THE RAILWAY TRANSITION SPIRAL

TALBOT

THE

Railway Transition Spiral

BY

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PREFACE

The railway transition spiral here presented is a flexible easement curve of general applicability and of comparatively easy analysis. The conceptions and methods used are similar to those of ordinary circular railroad curves. The definition is based upon degree-of-curve, and degree-of-curve, central angle, and deflection angle may be calculated, and the curve may be located by transit and chain or by co-ordinates from tangent and circular curve. The field work is quite similar to that of circular curves. The spiral is easily applied to a variety of field problems and to a wide range of location and old track conditions. In the principal formulas, angles, co-ordinates, offsets, etc., are expressed in terms of the length or distance along the spiral. The use of series in the development of the properties permits an estimate of the error involved in discarding negligible terms

The principal easement curves in use give alignments which approach each other very closely, so that, for equal easements, it may be expected that the riding qualities will not differ sensibly. In general, ease of calculation, simplicity of field work, and general applicability and flexibility will determine the form of easement curve to be selected. To establish the underlying principles of an easement curve of any range requires considerable mathematical analysis. The ordinary treatment of circular railway curves assumes previous knowledge of the geometrical properties of the circular curve, but the properties of the railway spiral must be deduced from the beginning. Fortunately the spiral is not complex, and its properties prove to be simple and general. A general treatment of such a curve has many advantages over approximate or special treatments. Approximate solutions may overlook important variables, and short methods may be limited to short and inefficient easements. The range of conditions of railway curves is so wide that it is best to develop methods of fairly general applicability, and these may then be simplified in meeting individual conditions.

The treatment herein given has been quite widely used on the railroads of the United States and many engineers have commended its simplicity, convenience, and flexibility. The methods and principles are readily taken up by instrument men, and the field work has proved little more difficult than that for circular curves. The use of a regular rate of transition per 100 feet of spiral is advantageous, and the tables are in convenient form.

The treatment of the railway transition spiral was published in Technograph No. 5, 1890-91, and was published in field-book size in 1899. Careful attention has been given in this revision to illustrative examples and explanations. The tables have been extended and a treatment of the Uniform Chord Length Method and of Street Railway Spirals added. For much of the latter, acknowledgment is made to Mr. A. L. Grandy. The writer is indebted to Messrs. J. K. Barker, Alfred L. Kuehn, and many others for valuable assistance in the preparation of tables and text.

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the utility of transition curves. The principles and some of the applications of one of the best of these curves, the railway transition spiral. will be considered.

2. Definition. *The Transition Spiral is a curve whose degree-of-curve increases directly as the distance along the curve from the point of spiral.*

Thus, if the spiral is to change at the rate of 10° per 100 feet, at 10 feet from the beginning of the spiral the curvature will be the same as that of a 1° curve; at 25 feet, as of a 2°30′ curve; at 60 feet, as of a 6° curve. Likewise, at 60 feet, the spiral may be compounded with a 6° curve; at 80 feet, with an 8° curve, etc.

This curve fulfills the requirements for a transition curve. Its curvature increases as the distance measured around the curve. The formulas for its use are comparatively simple and easy. The field work and the computations necessary in laying it out and in connecting it with circular curves are neither long nor complicated, and are similar to those for simple circular curves. The curve is extremely flexible, and may easily be adapted to the requirements of varied problems. The rate of change of degree-of-curve may be made any desirable amount according to the curve used, the maximum speed of trains, or the requirements of the ground.

As the derivation of the formulas is somewhat long, their demonstration will be given first. The explanation and application of these formulas to the field work and to the computations will be given separately, a knowledge of the demonstration not being essential to the application.

3. In Fig. 1, DLH is the circular curve and AP the prolongation of the initial tangent which are to be connected by the transition spiral. D is the point where the completed circular curve gives a tangent DN parallel to the tangent AP, and will be called the P. C. of the circular curve. AEL is the transition spiral connecting the

initial tangent AP with the main or circular curve LH. A is the beginning of the spiral and will be known as P.S., point of spiral. L is the beginning of the circular curve LH, and will be called P.C.C., point of circular curve. AP will be used as the axis of X, and A as the origin of co-ordinates. BD is the offset between the tangent AB of a circular curve and spiral, and the parallel tangent DN of an unspiraled curve.

The degree-of-curve of the spiral at any point is the same as the degree of a simple curve having the same radius of curvature as the spiral has at that point. The radius of the spiral changes from infinity at the P.S. to that of the main curve at the P.C.C. The spiral and a simple curve of the same degree will be tangent to each other at any given point; *i. e.*, they will have a common tangent.

4. The following notation will be used:

P.S. = Point of Spiral. (A, Fig. 1.)

P.C.C. = point where spiral compounds with circular. curve. (L, Fig. 1.)

P.C. = beginning of offsetted circular curve. (D, Fig. 1.)

R = radius of curvature of the spiral at any point.

D = Degree-of-curve of the spiral at any point; sometimes called D_1 at the end of spiral. Generally D_1 is made the same as D_0, the degree of the main curve.

a = rate of change of the degree-of-curve of the spiral per 100 ft. of the length. It is equal to the degree-of-curve of the spiral at 100 ft. from the P.S.

s = length in feet from the P.S. along the curve to any point on the spiral.

L = number of 100-ft. stations from the P.S. along the curve to any point on the spiral; in other words the distance to any point measured in units (or stations) of 100 ft. For the whole spiral (to P.C.C.) it is sometimes called L_1.

I = Total central angle of the whole curve (intersection angle), or twice BCH of Fig. 1, H being the middle of the circular arc.

Δ = angle showing the change of direction of the spiral at any point, and is the angle between the initial tangent and the tangent to the spiral at the given point. For the whole spiral it is equal to PTL and may be called Δ_1. The latter is also equal to DCL.

θ = spiral deflection angle at the P.S. from the initial tangent to any point on the spiral. For the point L (Fig. 1) it is BAL.

Φ = deflection angle at any point on the spiral, between the tangent at that point and a chord to any other point. At L, for the point A, Φ is TLA.

x = abscissa of any point on the spiral, referred to the P.S. as the origin and the initial tangent as the axis of X. For the point L, x = AM.

y = ordinate of the same point, measured at right angles to the above axis. For the point L, y = ML.

t = abscissa of the P.C. of the main curve produced backward; *i.e.*, of a simple curve without the spiral. For P.C. at D, t = AB.

o = offset between the initial tangent and the parallel tangent from the main curve produced backward, or it is the ordinate of the P.C. of the produced main curve. If D is the P.C., BD is o. It is also the radial distance between the concentric circles LH and BK.

T = tangent-distance for spiral and main curve = distance from A to the intersection of tangents.

E = external-distance for spiral and main curve.

C = long chord AL of the transition spiral.

u = distance along initial tangent from P.S. to intersection with spiral tangent = AT for point L.

v = length of spiral tangent to intersection with initial tangent = TL for point L.

5. The length of the spiral is to be measured along chords around the curve in the same way that simple curves are usually measured, using any length of chord up to a limit which depends upon the degree-of-curve of the spiral. The best railroad practice, in the writer's opinion, considers circular curves up to a 7° curve as measured with 100-ft. chords, from 7° to 14° as measured with 50-ft. chords, and from 14° upwards as measured with 25-ft. chords; that is to say, a 7° curve is one in which two 50-ft. chords together subtend 7° of central angle, a 14° curve one in which four 25-ft. chords together subtend 14° of central angle. The advantages

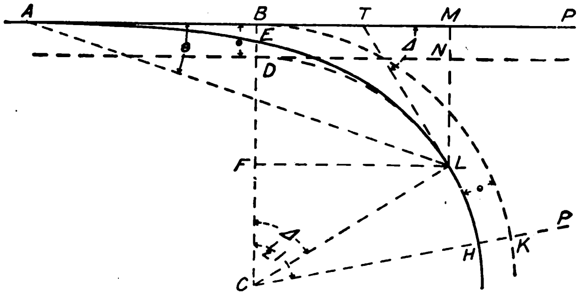

FIG. 1

of this method are two-fold,—the length of the curve as measured along the chords more nearly approximates the actual length of the curve, and the radius of the curve is almost exactly inversely proportional to the degree-of-curve. The latter consideration is an important one, simplifying many formulas. With this definition of degree-of-curve, the formula $R = \frac{5730}{D}$ will give no error greater than 1 in 2,500. For a 10° curve the error in the radius is .15 feet, and for a 16° curve .06 feet. This approximate value of R will give a resulting error in the length of the spiral, for the ordinary limits of spirals,

of less than $\frac{1}{6000}$ of the length, and will not reach 0.1 ft. The resulting error in alignment is $\frac{1}{6000}y$, and will not reach 0.01 ft. The difference between the length of the curve and that of these chords is less than 1 in 7000. For spirals measured with lengths of chords as here specified, or shorter, the error either in alignment or distance will be well within the limits of accuracy of the field work, and hence the relation $R = \frac{5730}{D}$ will be considered true.

THEORY

6. Intersection angle Δ.—From the definition of the transition spiral, we have, remembering that the value of a as defined above requires the length of curve to be measured in 100-ft. units (*stations*) instead of feet,

$$D = aL = \frac{a\,s}{100} \quad \ldots\ldots (1)$$

For the P.C.C. this becomes $D_1 = aL_1$.

From the calculus the radius of curvature

$$R = \frac{ds}{d\Delta}.$$

Substituting the expression $R = \frac{5730}{D}$ and solving,

$$d\Delta = \frac{a\,s\,ds}{573000}.$$

Integrating, $\Delta = \frac{a\,s^2}{1146000} = \frac{a\,L^2}{114.6}$.

Changing Δ from circular measure to degrees,

$$\Delta = \tfrac{1}{2}aL^2 \quad \ldots\ldots (2)$$

which is the intrinsic equation of the Transition Spiral.

For the P.C.C. this becomes $\Delta_1 = \frac{1}{2}\,a\,L_1^2$.

Since from (1) $a = \frac{D}{L}$, we also have

$$\Delta = \tfrac{1}{2}\,D\,L = \tfrac{1}{2}\frac{D^2}{a} \quad \ldots\ldots (3)$$

Since $L = \frac{D}{a}$

From these equations it will be seen that

(a) the change of direction of the spiral varies *as the square of the length* instead of as the first power of the length as in the simple circular curve, and

(b) the transition spiral for any angle Δ will be twice as long as a simple circular curve.

7. Co-ordinates x and y. To find the co-ordinates, x and y, of any point on the spiral, we have by the calculus $dy = ds \sin \Delta$ and $dx = ds \cos \Delta$. Expanding the sine and cosine into infinite series, substituting for ds its value in terms of $d\Delta$, and integrating, we have

$$y = \frac{1070.5}{(a)^{\frac{1}{2}}} \left\{ \tfrac{1}{3} \Delta^{\frac{3}{2}} - \tfrac{1}{42} \Delta^{\frac{7}{2}} + \tfrac{1}{1320} \Delta^{\frac{11}{2}} - \text{etc.} \right\} \quad (4)$$

$$x = \frac{1070.5}{(a)^{\frac{1}{2}}} \left\{ \Delta^{\frac{1}{2}} - \tfrac{1}{10} \Delta^{\frac{5}{2}} + \tfrac{1}{216} \Delta^{\frac{9}{2}} - \text{etc.} \right\} \ldots\ldots (5)$$

As Δ here is measured in circular measure and is only $\frac{1}{2}$ when the angle is 28°.65, these series are rapidly converging, especially for smaller angles.

Changing the angle Δ from circular measure to degrees, substituting for Δ the value given in (2), and dropping the small terms,

$$y = .291\ a L^3 - .00000158\ a^3 L^7 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For values of Δ less than 15° the last term may be dropped, and up to 25° the term will be small. $D L^2$ may also be written in place of $a L^3$. For all except extreme lengths, the last term may be dropped. Using $y = 291 aL^3$, it is seen that y varies as the cube of the distance of the point from the P.S.

Likewise changing Δ from circular measure to degrees etc., equation (5) becomes

$$x = 100\ L - .000762\ a^2 L^5 + .0000000027\ a^4 L^9 \ldots\ldots (7)$$

$$\text{Or } x = 100\ L - .000762\ D^2 L^3 \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The second term in second member of equation (7) or (8) may be used as a correction to be subtracted from the length of the curve in feet. The last term in equation (7) can be omitted, except for extreme lengths.

8. Spiral deflection angle θ.—It is desired to find the deflection angle θ for any point on the spiral, as BAL for the point L (Fig. 1). To show that this is nearly $\frac{1}{3}\Delta$, divide equation (4) by equation (5).

$\tan\theta = \frac{1}{3}\Delta + \frac{1}{105}\Delta^3 + \frac{26}{155925}\Delta^5$, etc. But from the tangent series for $\frac{1}{3}\Delta$,

$\tan\frac{1}{3}\Delta = \frac{1}{3}\Delta + \frac{1}{81}\Delta^3 + \frac{2}{3645}\Delta^5$, etc. Subtracting one from the other, we get a series which is rapidly decreasing when Δ is less than 40°. Investigating this difference, remembering that Δ is in circular measure, it is found that the error of calling the two equations equal is less than 1′ for $\Delta = 25°$ and decreases rapidly below this. As Δ will rarely reach 25°, and as the discrepancy is only a small fraction of a minute for any angle ordinarily used, and as the resultant error of direction will be corrected at the P.C.C. when $\Delta - \theta$ is turned off, we may ordinarily disregard this and write

$$\theta = \tfrac{1}{3}\Delta = \tfrac{1}{6}aL^2 = \tfrac{1}{6}\frac{D^2}{a} \quad \ldots\ldots\ldots\ldots (9)$$

where θ is in degrees.

From equation (9) it is seen that the spiral deflection angles to two points on the spiral will be to each other as the square of the distances to the points.

9. The error in equation (9) is dependent upon the value of Δ or θ and hence may be expressed independently of the length of spiral and rate of transition. For Δ between 20° and 40°, the number of minutes correction to be subtracted from $\frac{1}{3}\Delta$ or $\frac{1}{6}aL^2$ to give θ is $.000053\,\Delta^3$ where Δ is in degrees. The following table gives the deductions for various angles, and for other values interpolations may be made:

Correction in minutes to be subtracted from $\frac{1}{3}$ Δ or $\frac{1}{6}\ a\ L^2$ to give more precise values of θ.

Δ	Cor.	Δ	Cor.	Δ	Cor.
12°	0.1	21°	0.5	30°	1.4
15°	0.2	24°	0.7	33°	1.9
18°	0.3	27°	1.0	36°	2.4

Thus when Δ is 18°, the real value of θ will be 6°—0′.3 = 5° 59′.7. For a value of θ near 6° the same correction may be made. It will be seen that for the spiral de-

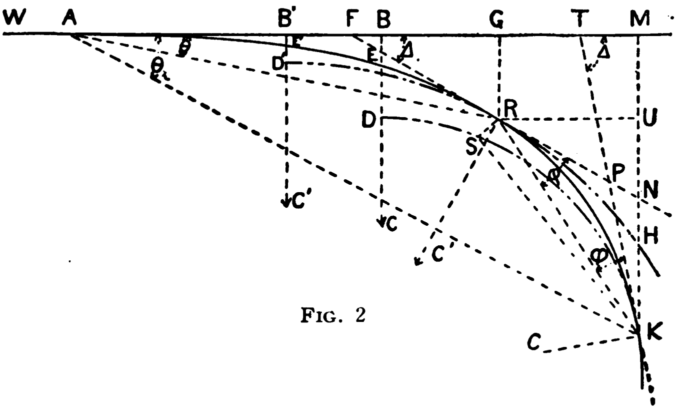

FIG. 2

flection angles ordinarily used the correction may be neglected without material error.

For the terminal-point of the spiral, the P.C.C., the value of θ_1 may be obtained from equation (9). In the extreme cases, where a further term is needed, the correction may easily be made from the above table.

10. Spiral tangent. To find the tangent at any point of the spiral, L, lay off a deflection angle from LA equal to Δ—θ. When Δ is not over 20°, $\frac{2}{3}$ Δ, or 2θ may be used. This since FLT = PTL = Δ, and FLA = PAL = θ. This is true for any point.

For the terminal point of the spiral, P.C.C., this becomes $\Delta_1 - \theta_1$ which is generally expressed with sufficient precision by $\frac{2}{3}\Delta_1$.

11. Deflection angle at point on spiral.—The deflection angle from the tangent at any point on the spiral to locate a second point may be found as follows: In Fig. 2, let L' be the distance from the P.S. to R, and L the distance from the P.S. to any other point on the spiral, as K. Let FRN be the tangent at R, and RFM $= \Delta'$ its angle with the initial tangent, and θ' the corresponding spiral deflection angle, RAF. Let KTM$=\Delta$ be angle of tangent at K with initial tangent, equal to total change of direction of the spiral up to that point. θ' and θ are the deflection angles at the P.S. for R and K respectively. KRN $= \Phi =$ required deflection angle. KRU $= \Phi + \Delta'$.

To show that angle $\Phi + \Delta'$ is almost exactly the same as the angle $\frac{1}{3}\dfrac{\Delta^{\frac{3}{2}} - \Delta'^{\frac{3}{2}}}{\Delta^{\frac{1}{2}} - \Delta'^{\frac{1}{2}}}$ or $\frac{1}{3}\left(\Delta + \Delta^{\frac{1}{2}}\Delta'^{\frac{1}{2}} + \Delta'\right)$, the following somewhat long and tedious operation may be gone through. It is thought not necessary to give it in detail the.

$$\tan\ (\Phi + \Delta') = \frac{\text{UK}}{\text{RU}} = \frac{y - y_1}{x - x_1}$$

Substitute for the co-ordinates in the above equation their values from equations (4) and (5), and also develop

$$\tan\ \frac{1}{3}\,\frac{\Delta^{\frac{3}{2}} - \Delta'^{\frac{3}{2}}}{\Delta^{\frac{1}{2}} - \Delta'^{\frac{1}{2}}}$$ into a series, and subtract the latter from the former. An expression for the difference will be found, which amounts to but a small fraction of a minute for any value of Δ up to 35°. Hence we may write

$$\Phi + \Delta' = \tfrac{1}{3}\left(\Delta + \Delta^{\frac{1}{2}}\Delta'^{\frac{1}{2}} + \Delta'\right)$$

By substitutng for Δ and Δ' their values in terms of L and L′ and reducing, the following value for Φ is found:

$$\Phi = \tfrac{1}{2}aL' \ (L - L') \pm \tfrac{1}{6}a \ (L - L')^2 \ldots\ldots\ldots\ldots (10)$$

Also, $\Delta' - \theta' \pm \Phi = \theta + \tfrac{1}{6}D'L$ ....(11)

And $\Delta' \pm \Phi = \theta' + \theta + \tfrac{1}{6}D'L$ ....(12)

Even for very large angles these equations are quite accurate if the exact value of θ is used. In equation (10) the last term $\tfrac{1}{6}a(L - L')^2$ should receive the same correction as an equal value of θ. Of course, for any angle ordinarily used no correction need be made. See correction for θ, page 9.

12. In equation (10) it will be noticed that the first term ($\tfrac{1}{2}$ a L' $(L—L')$) is equal to the deflection angle for a simple circular curve of the same degree as the spiral at the point R (*i.e.*, a L') and of a length equal to the distance between the two points; while the second term ($\tfrac{1}{6}a(L - L)^2$) is equal to the spiral deflection angle at the P.S. from the initial tangent for an equal length of spiral $(L—L')$.

If the point to be located had been chosen on the side of R nearer to the P.S., the two terms of equation (10) would have opposite algebraic signs, and the difference of the two quantities would be used. To show that the arithmetical difference of the two terms is to be used for a point nearer the P.S. when the distance $(L—L')$ is used without regard for the algebraic sign, equation (10) has been written with the plus and minus sign.

13. The spiral then deflects from a circle of the same degree-of-curve at the same rate that the spiral deflects from the initial tangent at the beginning. D′RH, in Fig. 2, represents the circular curve tangent to spiral at R, the two having the same radius at that point and both

being tangent to FRN. The deflection angles between points on the spiral and on the circle RH, and also between the spiral and RD′ are the same as the spiral deflection angle for an equal length of spiral from A. In the same way at K, RKT = SKT — SKR, the latter angle being equal to the deflection from initial tangent at A for a length of spiral equal to KR.

14. Equation (11) shows that the angle at any point between the chord joining this point with the P.S. and a chord to any other point (the angle, Fig. 2, between AR produced and RK if the point K is to be located from R) is equal to the spiral deflection angle at the P.S., θ, for the point to be located (KAM) plus one third of the deflection angle for a circular curve of the same degree as that of the spiral at the vertex of the angle, R, and of the length of the spiral from P.S. to the point K. This is true whether the point to be located is nearer the P.S., or farther, than the point used as the vertex of the angle.

It may also be readily shown from (2) that the difference in direction of the two tangents, $\Delta - \Delta'$, is the central angle for this simple curve plus the spiral angle, both for a length equal to the distance between the two points.

15. Equation (12) gives the value of the deflection angle from a line parallel to the initial tangent, the spiral deflection angle θ' for the point R being added to the values in equation (11).

16. Ordinates from osculating circle. It may also be shown that the offset distance between a point on the spiral and one on the osculating circle is the same as the ordinate y from the initial tangent at a point the same distance from the P.S. as the former point is from the point of osculation. These ordinates may be measured in direction normal to the circular curve.

17. Offset *o*.—From Fig. 1, BD = BF—DF = BF—CD vers DCL. But $o =$ BD, BF $= y$ for the end of spiral, DCL $= \Delta$ for the whole spiral, and CD $= R$. Hence, $o = y—R$ vers Δ. Substituting for y, R and Δ their values in terms of the length of the whole spiral, applying the versed sine series, and reducing we have for o in feet

$$o = .0727\ a\ L_1^3 = .0727\ D_1\ L_1^2 \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where D_1 and L_1 refer to the whole length of the spiral. The other terms of the series are so small that they may be dropped when Δ is less than 30°. The next term is $—.0000002\ a^3\ L^7$. It will be seen that o is approximately one fourth of the ordinate of the P.C.C., which, of course, should be true if E, the middle point of the spiral, is opposite D, the P.C.

18. Offset given. From (13) and (3) we have

$$L = 3.709\sqrt{\frac{o}{D}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$\Delta = 1.857\sqrt{o\ D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$a = .269\sqrt{\frac{D^3}{o}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

$3\frac{5}{7}$, $1\frac{6}{7}$ and $\frac{7}{26}$ may be used for these co-efficients with advantage.

19. Abscissa of P.C., *t*.—From Fig. 1, $t =$ AB $=$ AM —BM $= x$—FL $= x—R \sin \Delta$. Expanding and reducing,

$$\left.\begin{array}{l} t = 50\ L_1—.000127\ a^2\ L_1^5 \\ \text{or} \\ t = 50\ L_1—.000127\ D_1^2\ L_1^3 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

It should be noted that the full length of the spiral is used in the formula. The last term may be used as a correction to be subtracted from the half length of the spiral. It is easily tabulated for the principal spirals, and corrections for other spirals may be found by multi

plying the value with $a=1$ for the given length of spiral by the square of the a used.

20. A comparison of t with the abscissa found by substituting $\frac{1}{2} L_1$ in equation (8) shows that BD cuts the spiral at a point only .0001 $a^3 L^5$ feet from the middle point of the spiral. This is $\frac{4}{5}$ of the correction used in equation (17) for finding t from $\frac{1}{2} L_1$. For our purpose we may say that BD bisects the spiral. It also follows that the spiral bisects the line BD, since BE $= \frac{1}{8} y$. This is subject to slight error for large angles.

The length of the spiral from the P.S. to BD, therefore, exceeds t by one fifth of the t correction, and the remainder of the spiral exceeds the length of the circular curve from the P.C. to the P.C.C. by four fifths of the t correction. The entire length of the spiral exceeds the distance measured on t (AB, Fig. 1) plus the distance measured around the circular curve (DL, Fig. 1) by the t correction given in equation (17).

21. Tangent-distance T.—To find T, consider in Fig. 1 that AB intersects CH, H being the middle of the circular curve, at some point P outside the diagram. Then $\mathrm{T} = \mathrm{AP} = \mathrm{AB} + \mathrm{BP}$. $\mathrm{BP} = \mathrm{BC} \tan \mathrm{BCH}$.

$$\text{Hence } T = t + (R + o) \tan \tfrac{1}{2} I \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

t and $o \tan \frac{1}{2} I$ may be computed separately and added to the T found from an ordinary table of tangent-distances.

22. Equation (18) gives T for the same transition spiral at each end of the main curve. It may be desirable to make one spiral different from the other. To find an expression for the tangent-distance for this case proceed as follows: In Fig. 3, let $\mathrm{RS} = \mathrm{HD} = o_2$, $\mathrm{BD} = o_1$, $\mathrm{AB} = t_1$, $\mathrm{RT} = t_2$, $\mathrm{AE} = \mathrm{T}_1$, $\mathrm{TE} = T_2$, $R =$ radius of main curve DLKS, $\mathrm{R} + o_2 =$ radius of HR, and $I =$ angle PER.

Then $T_1 = t_1 + \text{HC} - \text{PE}$, and

$T_1 = t_1 + (R + o_2) \tan \frac{1}{2}I - (o_1 - o_2) \cot I$ . . . . (19)

Similarly, $T_2 = t_2 + (R + o_2) \tan \frac{1}{2}I + (o_1 - o_2) \operatorname{cosec} I.$

When I is more than 90°, the last term of (19) becomes essentially positive.

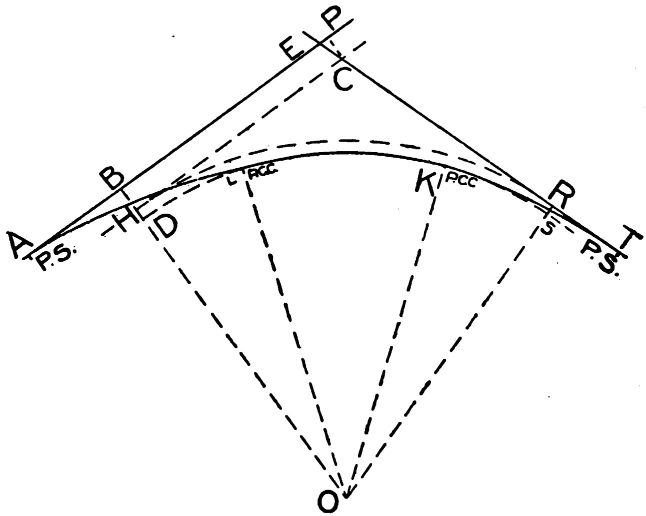

FIG. 3

23. External-distance E.—In Fig. 1, $E = \text{HP}$. HP $= \text{KP} + \text{HK}$. Hence

$$E = (R + o) \operatorname{exsec} \tfrac{1}{2} I + o \text{ . . . . } (20)$$

24. Long chord C.—In Fig. 1, $C = \text{AL}$. $\text{ML} = \text{AL} \sin \text{MAL}$, or $C = \frac{y}{\sin \theta}$. Putting this in terms of the length of the curve,

$$C = 100\, L - (.000338\, a^2L^5 \text{ or } .000338\, D^2L^3 \text{ . . . . . } (21)$$

in which C is in feet and L in stations. It will be seen that the last or correction term is four ninths of the correction for x as given in equation (7). When the cor-

rection term for x is tabulated or otherwise known, the length of the long chord may conveniently be calculated by subtracting four ninths of this x correction term from the whole length of the spiral.

The length of a chord which does not go through the P.S. may be calculated from the triangle formed by it and the two long chords drawn from its ends to the P.S. For all except extreme lengths, a chord may be taken as having the same length as the chord of an equal circular arc whose radius is the same as that at the middle point of the given spiral arc.

25. Spiral tangent-distances In. Fig. 1, $u = \mathrm{AT} = \mathrm{AM} - \mathrm{MT}$. As $\mathrm{MT} = y \cot \Delta$ or $v \cos \Delta$,

$$\left.\begin{aligned} u &= x - y \cot \Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ u &= x - v \cos \Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} (22)$$

Also $v = TL = \dfrac{y}{\sin \Delta.}$ Expanding $\sin \Delta$ into series, substituting the value of y from equation (6), and reducing,

$$v = \frac{y}{\sin \Delta} = \frac{100}{3} L + .000244\ a^2 L^5 \ldots\ldots\ldots\ldots (23)$$

The last term is almost exactly one third the corresponding term in equation (7), and hence v may be found by *adding* one third of the correction term used for determining x to one third of the length of the spiral in feet.

26. Middle ordinate. The middle ordinate for any arc of the spiral is equal to the middle ordinate for an equal length of circular curve of the same degree-of-curve as the spiral at the middle point of the arc considered. This degree-of-curve is the mean of the D's at the end of the given arc. This is an approximate formula which is true whether one end of the chord is at the P.S. or not.

The ordinate from any other point along a chord may be found as follows: Since the spiral diverges from the osculating circle at the middle point of the arc at the same rate as from the initial tangent, the amount of this divergence may be calculated by the method given on page 14 and added to or subtracted from the ordinate for the osculating circular curve. For a point nearer the P.S. than the center of the spiral arc, the divergence will be added to the ordinate of the circular arc, and for one farther away it will be subtracted from the ordinate. As before, the degree of the osculating circular curve is the mean of the D's at the end of the given arc.

Other properties may be found by ordinary trigonometric operations.

SUMMARY OF PRINCIPLES

27. For convenience of reference the principal formulas will be repeated here.

$$\left.\begin{array}{l} D = aL \text{ and } L = \dfrac{D}{a} \\ D_1 = aL_1 \text{ for whole spiral} \end{array}\right\} \quad (1)$$

$$\left.\begin{array}{l} \Delta = \tfrac{1}{2}aL^2 = \tfrac{1}{2}DL = \tfrac{1}{2}\dfrac{D^2}{a} \\ \Delta = \tfrac{1}{2}aL_1^2 = \tfrac{1}{2}D_1L_1 \text{ for whole spiral} \end{array}\right\} \quad (2)$$

$$y = .291\ aL^3 - \text{etc} \quad (6)$$

$$x = 100\ L - .000762\ a^2\ L^5 + \text{etc} \quad (7)$$

$$\left.\begin{array}{l} \theta = \tfrac{1}{3}\Delta = \tfrac{1}{6}aL^2 = \tfrac{1}{6}DL = \tfrac{1}{6}\dfrac{D^2}{a} \\ \theta_1 = \tfrac{1}{3}\Delta_1 = \tfrac{1}{6}aL_1^2 \text{ for whole spiral} \end{array}\right\} \quad (9)$$

$$\Phi = \tfrac{1}{2}aL' \ (L - L') \pm \tfrac{1}{6}a \ (L - L')^2 \quad \ldots\ldots (10)$$

$$\Delta' - \theta' \pm \Phi = \theta + \tfrac{1}{6} D'L \quad \ldots\ldots (11)$$

$$\Delta' \pm \Phi = \theta' + \theta + \tfrac{1}{6}D'L \quad \ldots\ldots (12)$$

$$o = .0727 \ aL_1^3 = .0727 \ D_1 L_1^2 \quad \ldots\ldots (13)$$

$$L_1 = 3.709\sqrt{\frac{o}{D_1}} \quad \ldots\ldots (14)$$

$$\Delta_1 = 1.857\sqrt{oD_1} \quad \ldots\ldots (15)$$

$$a = .269\sqrt{\frac{D^3}{o}} \quad \ldots\ldots (16)$$

$$t = 50L_1 - .000127 \ a^2L_1^5 \quad \ldots\ldots (17)$$

$$T = t + (R + o) \ \tan \tfrac{1}{2}I \quad \ldots\ldots (18)$$

$$E = (R + o) \ \text{exsec} \ \tfrac{1}{2}I + o \quad \ldots\ldots (20)$$

$$C = 100 \ L - .00034 \ a^2L^5 \quad \ldots\ldots (21)$$

$$\left.\begin{array}{l} u = x - y \cot \Delta \ \ldots\ldots \\ u = x - v \cos \Delta \ \ldots\ldots \end{array}\right\} (22)$$

$$v = \frac{y}{\sin \Delta} = \frac{100}{3} L + .000244 \ a^2L^5 \quad \ldots\ldots (23)$$

An inspection of the formulas and demonstrations will show the following properties of the transition spiral:

28. Degree-of-curve The degree-of-curve at any point on the spiral equals the degree at 100 feet from the P.S. multiplied by the distance along the spiral from the P.S. to the point (Eq. 1). This distance must be expressed in units of 100 feet (stations). Thus, if $a = 2$, at 100 feet from the P.S. the spiral will be a 2° curve; at 25 feet ($L = .25$) a 0°30′ curve; at 450 feet, ($L = 4.5$) a 9° curve. $\frac{100}{a}$ is the number of feet of spiral in which D changes one degree. Thus, for $a = 2$ the spiral increases

its degree of curve one degree for each $\frac{100}{2} = 50$ feet; for $a = \frac{2}{3}$ one degree for each $\frac{100}{\frac{2}{3}} = 150$ feet.

At the terminal point, the P.C.C., where the spiral connects with the main curve, D will sometimes be represented by D_1, and this should generally equal the degree of the circular curve D_0. The total length of the spiral will be $\frac{D_1}{a}$. If $a = 2$, a 6° curve would require a spiral 3 stations (300 feet) long.

29. Angle Δ.—The angle Δ between the initial tangent and the tangent at any point on the spiral (the change of direction corresponding to central angle of circular curves) (LTP, Fig. 1, page 5) in degrees equals (Eq. 2):

(*a*) One half of a times the square of the distance in 100-ft. stations from the P.S. to the point; thus if $a = 2$, for 300 ft. from P.S., $L = 3$, and $\Delta = \frac{1}{2} \times 2 \times 3^2 = 9°$. Or

(*b*) One half of the product of this distance L by the degree-of-curve of the spiral at the given point; thus at 300 ft. with $a = 2$, $D = 6°$, and $\Delta = \frac{1}{2} \times 3 \times 6 = 9°$. Or

(*c*) One half of the square of degree-of-curve at the point divided by a; thus at 300 ft. with $a = 2$, $\Delta = \frac{1}{2} \times \frac{6^2}{2} = 9°$.

For the same angle, then, the spiral is twice as long as a circular curve, and for the same length the angle is one half that for a circular curve whose D is the same as that at the end of the spiral.

30. Spiral deflection angle θ. The spiral deflection angle θ at the P.S. from the initial tangent to any point on the spiral, as PAL in Fig. 1, is $\frac{1}{3}\ \Delta$, or $\frac{1}{6}\ a\ L^2$. Thus, for a point 300 ft. from the P.S. ($L = 3$), if $a = 2$, $\theta = \frac{1}{6} \times 2 \times 3^2 = 3°$. If the result is wanted in minutes, since $\frac{1}{6} \times$

$60=10$, use 10 instead of $\frac{1}{6}$. For 105.4 ft. with $a=2$, $\theta = 10 \times 2 \times (1.054)^2 = 22'$. θ is also one third of the deflection angle for a simple curve of the same degree as the spiral at the given point. Thus, as above, the deflection angle for 300 ft. of 6° curve is 9° and $\theta = \frac{1}{3} \times 9 = 3°$.

These values are subject to slight corrections for Δ larger than 15° or 20° as explained in the derivation of the formula on page 9.

31. Tangent at point on spiral. The deflection angle at any point on the spiral between the tangent at this point and the chord to the P.S. (TLA in Fig. 1) is $\Delta - \theta$. This enables the tangent to be found. For Δ less than 15°, the value $\frac{2}{3}\ \Delta$ or $2\ \theta$ is sufficiently accurate. Thus, for the preceding example, with $a=2$, for the point 300 ft. from the P.S., this angle is $2\theta = 6°$.

32. Deflection angle at point on spiral. For deflection angles from a point on the spiral to other points on the spiral, the principle that the spiral diverges from the osculating circle (circular curve of same degree) at the same rate that the spiral deflects from the initial tangent is of service. The angles may be treated in three ways, as follows:

33. Angles from tangent. By equation (10) the deflection angle between the tangent at a transit point on the spiral and the chord to any other point on the spiral (as CBH, Fig. 4) is the sum or difference of two angles: (1) the deflection angle for a circular curve of the same degree as the spiral at the transit point for a length equal to the distance between the two points, and (2) the spiral deflection angle θ for a lengh of spiral equal to the distance between the two points. The latter angle is ***plus*** if the desired point is farther from the P.S., and ***minus*** if nearer, than the point from which the deflections are made.

Thus, if $a=2$ and the transit be at B (Fig. 4), 250 ft. from the P.S., the degree-of-curve at the transit point will be 5°, and the deflection angle CBH to set a point 150 ft. ahead will be the sum of 3° 45′, ($\frac{1}{2}$ of 150 ft. of 5° curve) and 45′, (the spiral deflection angle for 150 feet, $10\times2\times\overline{1.5}^{2}$) or 4°30′. For D, 150 ft. back, it would be 3° 45′ — 45′ = 3° 0′.

34. Angles from chord.—Likewise by equation **(11)** the angle CBE, Fig. 4, (deflection angle from chord to P.S.,) may be calculated by adding the spiral deflection angle θ for the point C (GAC) to $\frac{1}{6}$ the product of the degree-of-curve at B by the number of stations from the P.S. to C. For $a=2$ and the transit at B, 250 ft. from

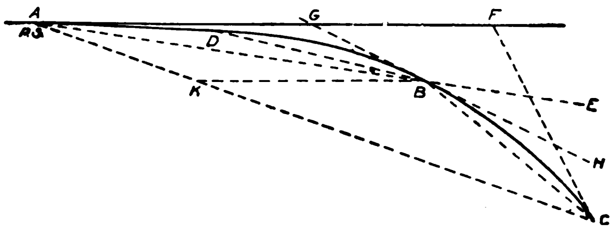

FIG. 4

the P.S., the degree-of-curve at the transit point is 5°, and the angle CBE to locate the point C 150 ft. ahead and 400 ft. from the P.S., will be ($\frac{1}{6}\times2\times4^2=5°20'$) + ($\frac{1}{6}\times5\times4=3°\,20'$) = 8° 40′. For the point D 100 ft. from the P.S., the angle DBA will be ($\frac{1}{6}\times2\times1^2=20'$) + ($\frac{1}{6}\times5\times1\cdot=50'$) = 1° 10′. This method is applicable whether the point to be located is nearer to, or farther from, the P.S. than the transit point. It permits the calculation of the spiral deflection angles at P.S. for the whole spiral and the determination of the angles between the chords in question by adding to these spiral deflection angles

$y=\frac{L^3}{L_1^3}y_1, =\left\{\frac{300}{400}\right\}^3\times 18.59=7.85.$ The deflection angle varies as the square of the distance and the ordinate as the cube of the distance from the P.S.

41. t and C.—The distance t from the P.S. to this offset (AB, Fig. 1) is found by subtracting the correction $.000127\ a^2\ L_1^5$ from the half length of the curve in feet. (Eq. 17.) Generally this correction term is quite small. As stated on page 13 this term may be tabulated, and it may also be obtained for a given length of spiral by multiplying tabulated values for $a=1$ by the square of the a of the given spiral. For this use see pages 25 and 26.

The long chord C is found by subtracting the correction, $.000338\ a^2\ L^5$, from the length of the curve in feet. (Eq. 21.) This correction may be found by multiplying the x correction for the same length of spiral by four ninths

42, u and v.—The spiral tangent-distances u and v (AT and TL, Fig. 1) are found by equations (22) and (23). v can be found most easily by taking one third of the tabulated values of the x correction and adding this to one third of the length of the spiral in feet.

THE TABLES

43. The computations may be shortened by the use the tables.

Tables I-XI gives the values of the principal parts of the transition spiral for the following values of a: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, 1, $1\frac{1}{4}$, $1\frac{2}{3}$, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 5, and 10. The column headed "Length" is the distance in feet along the spiral from the P.S. to any point on the spiral, and is equal to 100 times the L of the formulas. The column headed "x Cor."

in feet equals .291 times the product of a by the cube of the distance along the spiral from the P.S. to the point expressed in units of 100 ft. (stations). It therefore varies as the cube of the distance from P. S. Knowing y for one point, the y for a second point may be computed from it by this relation. $D\ L^2$ may be substituted for $a\ L^3$. For extreme lengths, a third term may have to be considered. As an illustration, with $a = 1$ for 200 ft. ($L=2$), $y = .291 \times 8 = 2.33$ ft. For 100 ft., y is one eighth as great; for 400 ft. y may be used as eight times as great, though the use of the next term of the series would change this somewhat.

39. Offset *o*.—The offset o between the initial tangent and the parallel tangent from the main curve produced backward, (BD, Fig. 1), in feet equals .0727 times the product of a by the cube of the length of the whole spiral in stations, or .0727 times the square of the length of spiral and the degree of main curve. This ordinate is approximately one fourth of the ordinate y of the end of spiral. The spiral bisects the offset at a point half-way between the P.S. and the P.C.C. (Eq. 11.) BE = ED. AE = EL. (Fig. 1.) The slight error in this is discussed in the derivation of the formulas (page 14). The value of o may best be discussed by means of one of the tables.

40. Calculation from known values.—When the length of the spiral is not so great that a second or correction term is needed for the values of θ, y, Δ, etc., it is seen from equations (9), (6), (2), etc., that these functions vary as the square and cube of the distance L and may be calculated from any known value. Thus if θ for 400 ft. is 2°40′, for 300 ft., $\theta = \frac{L^2}{L_1^{\,2}}\theta_1 = \left\{\frac{300}{400}\right\}^2 \times$ $(2°\ 40') = 1°\ 30'$. If y for 400 ft. is 18.59, for 300 ft.

$y = \frac{L^3}{L_1^3} y_1, = \left\{\frac{300}{400}\right\}^3 \times 18.59 = 7.85.$ The deflection angle varies as the square of the distance and the ordinate as the cube of the distance from the P.S.

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gives the correction to be subtracted from this distance in feet along the spiral to obtain x, and that headed "t Cor." gives the correction to be subtracted from the half length of the spiral in feet to obtain t. Both t Cor. and o are to be taken from the line for the full length of the spiral. For example, by Table IV, with $a=1$, to connect with a 5° curve, the length of spiral is 500 ft. and $L=5$; the change of direction Δ_1 is 12° 30′; the offset o to P.C. of circular curve is 9.07 ft.; t is $250-.4=249.6$ ft.; x is $500-2.37=497.63$ ft.; and the values of D, Δ, θ, y, and x Cor. for points 200, 210, 220 ft., etc., distant from the P.S. are found in the line with 200, 210, etc.

To find the long chord to P.S. C, subtract .445 of x Cor. from the length of the curve in feet. To find the spiral tangent-distance, v, add one third of x Cor. to one third of the length of the spiral in feet.

Tables I-IV have the values of Δ and θ calculated to the nearest tenth of a minute, and Tables V-VII to the nearest half minute. While this precision is not usually necessary, it may be of service where the sum of two or more angles is used.

44. Interpolation.—To find values intermediate between the distances given in the tables, interpolate by multiplying one tenth of the difference between consecutive values by the number of additional units. Thus, Table IV gives Δ for 400 ft. as 8° 00′; for 410 ft., 8° 24′.3. One tenth of the difference between these is 2′.4. For 406.8 ft., add $6.8\times 2.4=16'.3$ to 8°00′, giving 8°16′. For y, add $6.8\times .143=.97$ to 18.59, giving 19.56 ft. For o, add 6.8 times one tenth of .36 to 4.65 giving 4.89. D is 4.068 or 4° 4′.08.

Interpolation may also be made for other columns. Thus if o is given as 7.0 ft. and $a=1\frac{1}{4}$, by Table V the length of spiral will be between 420 and 430 ft. Interpolating, as o increases 0.5 ft. in 10 ft. of length, the .28

will be gained in 5.6 ft. and the length is 425.6 ft. Interpolation for Δ, D, etc., may then be made as before. Again, for $D = 4° 16'$, still using $a = 1\frac{1}{4}$, the length is between 340 and 350 and is $\frac{1}{7.5} \times 10 = 1.33$ ft. more than 340, making 341.33.

In general this interpolation gives accurate results and no correction need be made. For Δ the error in interpolation with values of a greater than 5 may need to be taken into account. To find exact values of Δ, deduct from the interpolated values a times the following quantities: For a length in feet ending with 1, .027′; 2, .048′; 3, .063′; 4, .072′; 5, .075′; 6, .072′; 7, .063′; 8, .048′; 9, .027′. It can easily be determined whether this correction need be considered. The difference arises from the fact that the square of numbers does not increase uniformly. For the other columns the errors of interpolation are very slight and may be neglected.

45. General use of Table IV.—Table IV has been carried to several decimal places to permit its use for values of a other than 1. To calculate values for another a, multiply the tabular value of D, Δ, θ, o, or y in Table IV for the distance from the P.S. to the point on the spiral by the a of the spiral used, and the x Cor. and t Cor. by the square of the a of the spiral. Thus if $a = 2.2$ and $L = 3.1$, multiply the D, Δ, θ, o, and y opposite 310 by 2.2, and the x Cor. and t Cor. by the square of 2.2. The values of y, o, and x Cor. obtained in this way are subject to slight errors for large values of a if Δ is more than 18°, but fortunately y for a distance greater than half of the length of the spiral is seldom needed, and as the error of this and the errors in o and x Cor. are ordinarily small the correction may generally be neglected. The amount of this error may be found by the method given in a succeeding paragraph. The error in θ is discussed on page 9.

To use Table IV for another a, it may be desirable 'st to determine the length of the spiral by dividing the $_1$ of the required spiral by a or to determine it from o. hus, for $a = 1.5$, to connect with a 6° curve, divide 6 r 1.5, which gives $L_1 = 4$; that is, the whole spiral will : 400 ft. long, and the properties for the spiral may be omputed by multiplying those in the line with the reiired distance by 1.5. In other words it must be borne mind that the distances in the column of lengths main unchanged with new values of a, and the quanies in all the other columns will be changed for a other an 1

46. Corrections for calculations.—For the calculaon of tables and other work requiring the recognition a further term in the equations, the value of the second term of the o series (.0000002 a^3 L^7, eq. (13)) and the second term of the y series (.00000158 a^3 L^7, eq. 3)) may be obtained by multiplying the quantities in e following table by a^3; and the third term of the x ries (.00000000268 a^4 L^9, eq. (7)) by a^4. These terms r o and y are negative, and the term for x is to be btracted from the x Cor.

L	o	y	x
2.50	.....	.0010	.....
3.00	.0004	.0035	.....
3.50	.0013	.010	.....
4.00	.0032	.026	.0007
4.50	.0074	.059	.002
5.00	.015	.124	.005
5.50	.030	.241	.012
6.00	.055	.442	.027
6.50	.097	.775	.055
7.00	.163	1.301	1.08

For making corrections on results obtained from Table r for a other than 1, subtract from the product of

the multiplication used to obtain the desired distance $a\ (a^2 - 1)$ times the value from the above table in obtaining o and y, and $a\ (a^3 - 1)$ times the value from the table in obtaining the x Cor.

47. Table of ordinates.—By Table XII the ordinate from the tangent or from the circular curve at a decimal part of the half length of the spiral may be obtained by the multiplication of o of the spiral by the factor given in the table. See method by co-ordinates and Fig. 5. It should not be forgotten that Tables I-X give ordinates, and that values for intermediate points may easily be interpolated.

48. Table of offsets.—Table XX gives values of o and L for various values of a. Within reasonable limits it will bear interpolation, both for intermediate values of a and D and to determine a for intermediate values of o. It is of service in location problems.

Tables XIII and XIV are described under Uniform Chord Length Method. The tables for street railway curves are described under Street Railway Spirals.

CHOICE OF *a* AND LENGTH OF SPIRAL.

49. The selection of a and with it the length of spiral require consideration. The value of a to be used is dependent upon the speed of trains, the maximum degree-of-curve, the length of tangents, the permissible offset of the line for the topographical conditions in question, the distance in which the superelevation of the outer rail may be attained, etc., and hence is subject to a wide range of conditions. It may, however, aid the engineer's *judgment to discuss* these conditions briefly.

50. Effect of speed. For the same rolling stock and for the same comfort in riding, it would seem that a given amount of superelevation must be attained in the same length of time; and hence it is probable that a should vary nearly inversely as the cube of the speed of the train. This conclusion also emphasizes the desirability of spiraling curves used under high speeds.

Assuming that $a=1$ is a proper value for speeds of 50 miles per hour, this principle would suggest the following maximum values of a; 60 miles per hour, $\frac{1}{2}$; 50 miles per hour, 1; 40 miles per hour, 2; 30 miles per hour, $3\frac{1}{3}$; 25 miles per hour, 5; 20 miles per hour, 10. While for the very high speeds this may seem to require unnecessarily long spirals and for low speeds short spirals, yet $a=1$ has given satisfactory results at speeds of 50 to 60 miles an hour, and $a=2$ at 40 to 50 miles an hour, and for 60 miles an hour, $a=\frac{1}{2}$ is not too small. Of course, in any case, longer spirals and smaller values of a will give smoother riding curves.

51. The speed of trains may be limited by the maximum superelevation allowable on the sharper curves. Under usual practice the requirement of maximum superelevation would limit the maximum degree-of-curve for speeds of 60 miles an hour to 3°, for 50 miles to 4°, for 40 miles to 6°, for 30 miles to 12°, etc. Where the track is not used for slow trains and a superelevation of more than 7 or 8 inches is allowable, somewhat higher speeds on such curves may be used. The maximum speed of train, however, will be the governing consideration in the choice of a rather than the maximum degree-of-curve.

52. Attainment of superelevation.—The rate of attaining the superelevation is sometimes given as the governing consideration, but in reality this rate is gov-

erned by the speed. The distance in which the outer rail should attain an elevation of 1 inch will not be the same for a speed of 60 miles an hour as for one of 40 miles. The schedule of maximum values of a for various speeds as given above involves, approximately, attaining 1 inch of elevation in the following distances: 60 miles, 80 feet; 50 miles, 53 feet; 40 miles, 44 feet; 25 miles, 40 feet. The best rate also depends upon stiffness of car springs, weight and style of cars, and other conditions. Generally speed and amount of superelevation should govern the length of spiral, and rate of attainment is subordinate.

53. It may be convenient for maintenance-of-way work to arrange the spiral so that the superelevation is attained at a definite rate per 100 ft. of length of spiral. Let k be this rate, expressed in inches of superelevation attained in 100 feet. Let h be the superelevation in inches per degree of curve; for a 3° curve, $3h$, etc. Then

$$k = ah = \frac{D}{L}h. \qquad a = \frac{k}{h}.$$

54. The following table shows the values of a, which gives rates of 1, $1\frac{1}{2}$ and 2 inches of superelevation attained in 100 ft. for the amount of superelevation per degree of curve given at the head of the columns.

VALUES OF a

Elevation per degree	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$
a for k equal to 1 in. per 100 ft.	2	$1\frac{1}{3}$	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$
a for k equal to $1\frac{1}{2}$ in. per 100 ft.	3	2	$1\frac{1}{2}$	$\frac{6}{5}$	1	$\frac{3}{4}$	$\frac{3}{5}$
a for k equal to 2 in per 100 ft.	4	$2\frac{2}{3}$	2	$1\frac{3}{5}$	$1\frac{1}{3}$	1	$\frac{4}{5}$
Velocity in miles an hour corresponding to superelevation	26.9	33.0	38.1	42.6	46.6	53.8	60.2

The length of spiral for $a = \frac{1}{2}$ is 200 ft. for each degree of curve; for $a = \frac{4}{5}$, 125 ft.; for $a = \frac{2}{3}$, 150 ft., etc. When

tables are not given for the a used, the values may be calculated from the tables by multiplication or other process. Thus for $a = 1\frac{1}{3}$, double the values for $a = \frac{2}{3}$; for $a = \frac{3}{4}$, multiply those from $a = \frac{1}{2}$ by $1\frac{1}{2}$ or those from $a = 1$ by $\frac{3}{4}$.

55. The amount of superelevation per degree of curve here used is calculated from .00069 V^2, where V is the velocity in miles per hour. This gives the number of inches per degree to counteract the centrifugal force, and is based on distance from center to center of rail. This amount is used here because it is a value quite commonly quoted; for other superelevations comparisons may readily be made with the figure here given. There is a wide divergence of opinion on the proper amount. A superelevation somewhat less than that required to counterbalance the centrifugal force produces a moderate flange pressure on the outer rail and is held by many to give a smoother riding track. Care must be taken that track so elevated is never used at speeds so far above the assumed speed as to be unsafe. For convenience in maintenance-of-way work it may be desirable to establish the superelevation at a convenient amount near the value calculated for the assumed velocity, an allowable practice since the assumed velocity may not be realized. Thus 2 inches per degree may be used in place of $1\frac{7}{8}$, etc.

56. Minimum spirals. For a given value of a there may be a question as to how flat a curve may profitably be spiraled. The spiral should certainly vary enough from the position of a simple circular curve that the distinction may not be obliterated by the inaccuracies of track work; otherwise it will be as advantageous to begin the superelevation an equal distance back on the tangent. The limits given in a former edition have been criticised by engineers of maintenance of way, and experience on

prominent roads indicates that the minimum limit of o there set, 0.6 to 1 ft., was too high. It seems that the gradual change of direction between the trucks and the car body, and the fitting of elevation to the curvature by the spiral, are advantageous to smooth riding even when the change in alignment is slight. Experience seems to indicate that for $a=\frac{1}{2}$ curves above 30′ may be spiraled; for $a=1$, 1° and above; for $a=2$, 2°; for $a=3\frac{1}{3}$, 3°; for $a=5$, 4°; for $a=10$, 6°. For curves lighter than these any advantage seemingly found by spiraling would probably be obtained by beginning the superelevation back on the tangent.

In any case decreasing the value of a and thus increasing the length of the spiral will increase the efficiency of the spiral and better the riding qualities of the curve. This view needs emphasizing, and too much should not be expected of short spirals.

57. Selection of *a* and length of spiral.—The selection of a, then, must be a matter to be left to the judgment of the engineer. As a guide the following table containing values of a which have given satisfactory results at the speeds noted is given. Lower values of a are of course advantageous; as, for example, at a speed of 40 miles an hour $a=1\frac{1}{3}$, or even 1 will make a more efficient easement than the one given. Higher values of a—shorter spirals—may be necessary in many cases, but it must be understood that they will not be so satisfactory. The column headed "Minimum curve spiraled" is the lightest curve which it is considered desirable to spiral with the value of a given opposite. "Maximum curve" is fixed at the given speed by the limit of superelevation; at lower speeds this a may profitably be used for sharper curves. The speeds are given in miles per hour and the elevations in inches

MINIMUM SPIRAL FOR MAXIMUM SPEED

Maximum Speed	a	Maximum Curve	Min. Curve Spiraled	Length per Degree	Elev. per Degree
60	$\frac{1}{2}$	3°	30′	200	$2\frac{1}{2}$
50	1	4°	1°	100	$1\frac{7}{8}$
40	2	7°	2°	50	$1\frac{1}{8}$
30	$3\frac{1}{3}$	11°	3°	30	$\frac{5}{8}$
25	5	14°	4°	20	$\frac{1}{2}$
20	10	25°	5°	10	$\frac{28}{100}$

For shorter spirals, the following values of a are consistent with each other: 60 miles per hour, 1; 50 miles per hour, $1\frac{2}{3}$; 40 miles per hour, $3\frac{1}{3}$; 30 miles per hour, $6\frac{2}{3}$; 25 miles per hour, 10.

LOCATION OF P.S., P.C.C., AND P.C.

58. Location from intersection of tangents.—When the tangents have been run to an intersection, the P.S. (A, Fig. 1, page 5) may be located by measuring back on the tangent from the point of intersection a distance equal to the tangential distance T (equation 18). This distance may also be computed by adding $t + o \tan \frac{1}{2}I$ to the tangential distance of the circular curve as ordinarily calculated. (See section 21.). The P.C. (D, Fig. 1) may be located by calculating o and offsetting this amount at a point on the tangent distant t from the P.S. (See section 19.). t may be found by subtracting the t COR. of the tables from the half length of curve in feet. Thus, for 400 ft. of spiral with $a = 2$, by Table VII t cor. is .5 ft. and $t = 200 - 0.5 = 199.5$ ft. The P.C.C. (L, Fig. 1) may then be located by running the spiral from the P.S., or by locating the circular curve from the P.C. for a distance $\frac{1}{2}L_1$.

59. Location from P.C. of a curve without spiral. In case a simple curve has been run without provision for a spiral and without offsets, that is in the usual way, it will be necessary to change the position of the circular curve. The distance of the P.S. back of the P.C. of the old simple curve will be $t + o \tan \frac{1}{2} I$, I being the total intersection angle. The new curve will come inside the old but will not be exactly parallel to it.

60. Location from P. C. of offsetted curve. If a simple curve has been run for use with spiral, as DLH in Fig. 1, page 5, o may be computed, the offset measured to B and the distance t (AB) measured to locate the P.S. (A). The length $\frac{1}{2} L_1$ measured from the P.C. on the circular curve will locate the P.C.C. (L). Similarly if the tangent is fixed, the curve may be located by first making the offset from the tangent to the P.C.

61. If both P.C. and tangent are fixed with an offset $o = \text{BD}$ between them, a may be found from $a = .269\sqrt{\frac{D^3}{o}}$ or a and L may be found from Table XX. After finding t, the P.S. may be located in the usual manner. For a 5° curve with $o = 10$ ft., by equations (14) and (16) $L = 525.3$ ft. and $a = .952$. Since with $a = 1$ and this length of spiral t cor. $= .5$, the correction to be used here is $0.5 \times a^2$, and $t = 262.65 - .45 = 262.2$ ft. This method is a great convenience where it is desired on account of the ground to throw the curve in or out without changing tangent, or where a similar change in the tangent is desired without a change in the curve, the connection to be made by means of a suitable spiral.

LAYING OUT THE SPIRAL BY CO-ORDINATES.

62. With the initial tangent as axis of X and the P.S. as the origin of co-ordinates, it is not difficult to locate points on the spiral by means of co-ordinates. These may be calculated from equations (6) and (7), or they may be taken from the tables. Beyond B of Fig 1, page 5, the ordinates become large and the x correction may be considerable. For long spirals, the second term of the y series, may need to be considered. The property that the spiral diverges from the circular curve at the same rate as from the initial tangent is of service. Between E and the P.C.C. (L) measure the ordinate or offset from the circular curve, using for this offset at a point a given distance from the P.C.C. the ordinate y of the spiral from the tangent at the same distance from the P.S. Thus for $a=1$, by Table I, for a point 200 ft. from the P.S., $y=2.33$ ft. To locate a point on the spiral 200 ft. from the P.C.C., offset from the circular curve this same distance, 2.33 ft.

63. Knowing the y for any point on the curve, the y for any other point within ordinary limits may be found by multiplying the former number by the cube of the ratio of the distances from the P.S. to the respective points; thus, if for a point 250 ft. from the P.S., $y=4.54$ ft., y at 300 ft. equals $(\frac{30}{25})^3 \times 4.54 = 7.85$ ft. Similarly, points may be located by offsets from the circular curve, the distance being measured from the P.C.C.

64. If the length of the half spiral be divided into an integral number of parts (See Fig. 5), any ordinate from the tangent or from the circular curve may easily be calculated from o ($o=.0727\,DL^2$, pp. 22 and 25), by mul-

tiplying *o* by one half the cube of the ratio of the number of parts this point is from the P.S. to the whole number of parts. The following table gives the factor by which the *o* of the spiral may be multiplied to determine the *y* of the point when the length of the half spiral is divided into 10 parts.

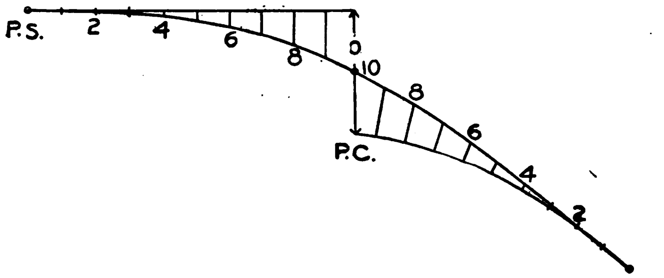

FIG. 5

TABLE OF FACTORS FOR ORDINATES.

To find *y*, multiply *o* by the factor.

Ratio to half length.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	**1.0**
Factor.	0005	.004	.014	.032	.063	.108	.172	.256	.365	**.500**

As an example, with $a=1$, *o* for a 5° curve (spiral 500 feet long) is 9.07 ft. The half length of the spiral is 250 ft. and one tenth of this distance is 25 ft. *y* at 100 ft. (0.4 of the half length) is $.032 \times 9.07 = .29$ ft. Similarly, .29 ft. will be the offset from the circular curve at a point 100 ft. from the P.C.C.

To divide the half-length of spiral into 5 parts and the whole spiral into 10 parts, use the even numbered tenths of the above table.

For intermediate values, interpolation in the above table will give reasonably accurate results. This enables interpolation for quarter points, and other fractional parts; thus, for .67 of half length the factor is .153.

The results by the above table are subject to error in the hundredths place, but for usual cases are within .02 ft.

65. Still another method is to measure ordinates from the initial tangent for about two thirds of the length of the spiral and for the remaining distance to measure ordinates or offsets from the terminal spiral tangent (TL, Fig. 1, page 5). The offsets from the terminal spiral tangent will be the difference between the offset for the produced circular curve DL and the y for a spiral, both for a length equal to the distance from P.C.C. to the point to be located. As this distance will be less than one third the spiral length, the approximate formula for tangent offset, $.873\ D_1\ L^2$, may ordinarily be used. D_1 is the degree of the spiral at the P.C.C. and L here will be used as the distance from the P.C.C. to the desired point. The offset will then be $.873\ D_1\ L^2 — .291\ a\ L^3$. While the offsets are longer than those from the circular curve, the measurements are made from the tangent and the circular curve need not be run. As an example take $a = 1$, and $D_1 = 4°$. Length of spiral is then 400 ft. For a point 50 ft. back of P.C.C. ($L = .5$) the offset is $.87 — .04 = .83$ ft. For 100 ft. from P.C.C. the offset is $3.49 — .29 = 3.20$ ft. This is a very convenient method.

66. Many engineers prefer the co-ordinate method. The circular curve is run from the P.C. established by making the offset from the initial tangent, and the spiral is then located by setting off ordinates from the simple curve between P.C. and P.C.C. and by ordinates from the initial tangent back to the P.S., or for the latter portion

by laying off $o—y$ from a tangent at the P.C. parallel to the initial tangent, the ordinates being calculated by one of the preceding methods; or offsets from the terminal spiral tangent may be made for the last third of the spiral length. This method is particularlly applicable to location work and to short spirals, though under many conditions it may readily be applied to setting track centers.

LAYING OUT THE SPIRAL BY TRANSIT AND DEFLECTION ANGLES.

67. The spiral may be run in with the transit by turning off deflection angles and making measurements along chords in much the same manner as circular curves. The deflection angles are easily calculated, and the field work is not more difficult than for circular curves. The ordinary transit-man will find no difficulty in understanding the work. Since it is not necessary to keep succeeding chords the same length as the first, the stationing may be kept up, and the even stations, +50's, and other points put in as usual. Herein is an advantage over methods requiring a regular length of chord to be used.

68. Transit at P. S. With the transit at the P.S., which has been located by one of the methods previously described, the deflection angle θ (BAL, Fig. 1, page 5) will locate points on the spiral. θ may be taken from the tables, or it may be calculated from equation (9), $\theta = \frac{1}{3} \Delta = \frac{1}{6} a L^2$. For this calculation, if desired, the square of L may be taken from a table of squares, the lower decimals dropped, and the multiplication by the simple factors remaining may be made easily and rapidly. Thus, when $a = 2$, to determine θ for a point 234 ft. (2.34 stations) from the P.S., find the square of 234 (54756), change the

decimal point so that it will become the square of 2.34 (5.48), and $\theta = \frac{1}{6}\ a\ L^2 = \frac{1}{6} \times 2 \times 5.48 = 1°\ 49'$. If the result is wanted in minutes, since $\frac{1}{6} \times 60 = 10$, use 10 instead of $\frac{1}{6}$. The slide rule may be used with advantage.

For a tabulated spiral, the spiral deflection angles may be taken from the table. Thus, for $a = 1\frac{1}{4}$, by Table V θ for 110 ft. is 15′. For 114 ft. interpolate proportionally between the tabulated value of θ for 110 and that for 120 ft. giving $14\frac{1}{2}'$.

If it is not desired that the even stations be located, the spiral may be located by 50-ft. chords, or chords of other length, directly from the P.C. and the labor of calculation will be reduced.

69. Transit on spiral. With the transit on an intermediate point on the spiral, the tangent to the spiral at this point may be obtained by turning off from the chord to the P.S. as a back-sight the angle $\Delta - \theta$ (ARF, Fig. 2, page 9), where Δ is the spiral intersection angle and θ is the spiral deflection angle at the P.S. for the given transit point (R). Except for extreme lengths this is equal to 2θ. Thus, by Table I ($a = \frac{1}{2}$) for a transit point 400 ft. from the P.S., the required angle is $4° - 1°\ 20' = 2°\ 40'$ (or 2 (1° 20′)).

70. For deflection angles from an intermediate transit point on ordinary circular curves, three methods are in use among engineers:

(a) The measurement and record of the angle between the tangent to the curve at the transit point and the chord to the point to be located.

(b) The use of the angle between the chord connecting the transit point to the P.C., and the chord to the point to be located.

(c) The use of the angle between a line through the transit point parallel to the initial tangent and the chord to the point to be located. Three corresponding methods may be used with the spiral and will be treated separately.

71. Intermediate deflection angles. (a) From tangent. By equation (10), the angle between the tangent at the transit point and any chord (as CBH, Fig. 4, page 21) is $\Phi = \frac{1}{2}aL'(L - L') \pm \frac{1}{6}a(L - L')^2$. See also pages 10 and 20. This method then involves the following steps: With transit at B (Fig. 4, page 21) set vernier at $\Delta' - \theta'$, or $2\theta'$ (these being the angles for transit at the P.S. for the point B), and back-sight on the P.S. so that the zero reading will give the tangent BH. To locate any point C find the sum of (1) one half of the product of the degree-of-curve at the transit point B by the distance in stations from the transit point to C and (2) the spiral deflection angle θ for the same distance. For D, find the difference of these quantities. Thus, for $a=1$, with the transit 300 ft. from the P.S., D' the degree-of-curve at B is 3°. By Table IV, θ' $(L = 3)$ is 1° 30′, and ABG is 3°, giving the position of the tangent at B. For C 100 ft. from B, add $\frac{1}{2} \times 3 \times 1 = 1° \, 30'$ and $\frac{1}{6} \times 1 \times 1^2 = 10'$, giving 1° 40′ for CBH. For D 100 ft. from B, DBG $= 1° \, 30' - 10' = 1° \, 20'$.

72. (b) From chord to the P. S. This is the method generally to be recommended. By equation (11) the angle between the chord from transit point to P.S. and any chord (as CBE, Fig. 4, page 21) is $\theta + \frac{1}{6} D' L$, θ being the spiral deflection angle from the initial tangent for the point to be located, D' the degree-of-curve at the transit point, and L the distance in stations from P.S. to the point to be located. (See also pages 10 and 21.) This method involves the following steps: With transit at B and vernier reading zero, back-sight on the P.S.

To locate C turn off an angle equal to the sum of (1) the spiral deflection angle θ for a distance equal to the distance from C to the P.S. and (2) one sixth of the product of the degree-of-curve at the transit point and L for the point C. Thus for $a = 1$, with the transit 300 ft. from the P.S., D′ at B is 3°. For C 100 ft. from B and 400 ft. from the P.S., add $\frac{1}{6} \times 1 \times 4^2 = 2° 40'$ (which is the spiral deflection angle θ for 400 ft.) and $\frac{1}{6} \times 3 \times 4 = 2°$ (which is $\frac{1}{6} D' L$) giving 4° 40′ for CBE. For D 100 ft. from B, DBA $= 40' + 1° = 1° 40'$.

To facilitate the calculation the transit point may be chosen at a point where the spiral has an even degree-of-curve, as in the above example, but this is not essential. It may be seen that $\frac{1}{10} D'$ gives the minutes per foot in $\frac{1}{6} D' L$.

73. (c) Angles with initial tangent. The use of angles with the line parallel to the initial tangent (BK, Fig. 4) is the same as (b) except that θ', the spiral deflecion angle to the transit point, must be added to all angles. Otherwise the method is the same as (b). Use equation (12).

74. Transit at P. C. C. With the transit at the P. C.C., the tangent to the curve may be found by turning off from the chord to the P.S. an angle $\Delta_1 - \theta_1$, these being the angles for the full spiral. Within ordinary limits this angle equals $2\theta_1$. The main circular curve may be run as usual.

In case the P.S. can not be seen from the P.C.C., the chord to the P.S. may be located by turning off from the chord to an intermediate point on the spiral an angle $\Delta_1 - \theta_1 - \Phi$ (ACB, Fig. 4, page 21) where Φ is the angle between the chord and the tangent at P.C.C. (BCF). (See page 10.)

To locate the chord from P.C.C. to P.C. (not shown in

any diagram), deflect from the chord to the P.S. the angle $\frac{1}{2}\Delta_1 - \theta_1$. To locate chord to P.C. from a chord to an intermediate point on spiral, deflect from chord to the intermediate point the angle $\frac{1}{2}\Delta_1 - \Phi$. With the data already at hand, it may be easier to calculate this angle as $\theta + \frac{1}{6} D' L - \frac{1}{2}\Delta_1 + \theta_1$, remembering that θ and L refer to the intermediate point and D', Δ_1, and θ_1 to the P.C.C.

75. For the circular curve some engineers prefer to measure the deflection angles from the tangent at the P.C.C., and others prefer to measure from the chord from P.C.C. to P.C. and thus maintain the same notes as though the spiral had not been used. By the use of the angles discussed in preceding paragraphs, either method may be used.

76. To run from the P.C.C. toward the P.S. Two methods may be used, (a) using L as the distance from P.C.C. and deflecting from the tangent, and (b) using L as measured from the P.S. and deflecting from chord to P.S.

77. (a) **Angles from tangent.** Using the distance L as measured from the P.C.C., deflect from the tangent to the curve an angle equal to the difference of (1) one half of the product of D_1 (degree-of-curve at P.C.C.) and distance L to point (which is the same as the deflection angle for $D_1°$ circular curve) and (2) spiral deflection angle θ for distance L, ($\frac{1}{6}\, a\, L^2$). This is the same as method (a) of "Transit on Spiral." The method depends upon the principle that the spiral deflects from the osculating curve at the P.C.C. at the same rate that it deflects from the initial tangent at P.S.

As an example take 400 ft. of spiral connecting with a 4° curve ($a = 1$). Measure L from P.C.C. For a point 150 ft. from P.C.C. ($L = 1.5$), take the difference between

$\frac{1}{2} \times 4 \times 1.5 = 3°$ and $22\frac{1}{2}'$ (spiral deflection angle for 150 ft.) which is $2°\ 37\frac{1}{2}'$. This angle is to be deflected from the tangent at P.C.C. By the same method the angle to locate the P.S. is $\frac{1}{2} \times 4 \times 4 = 8°$ minus $2°\ 40'$, or $5°\ 20'$, the result found by the usual method.

78. (b) **Angles from chord to P. S.** Using the distance L as measured from the P.S., deflect from the chord to the P.S. an angle equal to the sum of (1) the spiral deflection angle θ for distance L from the P.S. ($\frac{1}{6}\ a\ L^2$) and (2) one sixth of the product of D_1 (degree-of-curve at the P.C.C.) and L, ($\frac{1}{6}\ D_1\ L$). This is the same as (b) of "Transit on Sprial."

Using the example cited in the preceding method, 150 ft., from P.C.C. will be 250 ft. from P.S., and $L = 2.5$. $\theta = \frac{1}{6} \times 1 \times (2.5)^2 = 1°2\frac{1}{2}'$. $\frac{1}{6}D_1L = \frac{1}{6} \times 4 \times 2.5 = 1°40'$ The sum of these is $2°\ 42\frac{1}{2}'$, the angle to be deflected from the chord to P.S. By the same method, for P.S. L is 0, and the deflection angle proves to be 0, as it should be.

STA.	POINTS	θ	$\frac{1}{6}$D′L	
+29 ⊙	PCC. 8°	5°20′	3°37′ [8°57′]	Set Vernier at φ = 4°36′; back-sight on 19, and 0° reading gives tangent.
20		4°35′	3°21′ [7°56′]	
+50		3°26′	2°54′ [6°20′]	
19 ○	D′ = 5°42′	2°27′		Set Vernier at 0°, back-sight on P.S. and turn off angle in brackets
+50		1°37′		
18		0°58′		
17		0°10′		Curve to Right
16 + 29●	P.S.	0°0′		$a = 2$, $L_1 = 4$, $\Delta_1 = 16°$

FIELD NOTES

79. Transit notes. For a spiral with $a = 2$ connecting with an 8° curve, $L = 4$, and if the P.S. has been found to be at 16 + 29, the notes may be made as follows, using method (b) for the transit on the spiral. At Sta. 19 the

deflection angle from the chord to the P.S., as a back-sight, is the sum of those given in third and fourth columns, and it is here inclosed in brackets.

APPLICATION TO EXISTING CURVES

80. When a road has been constructed without transition curves, the ordinary application of the preceding principles will require a new line to be built inside the old curve, and the cost of construction may be considerable. To retain as far as possible the old roadbed, three methods are applicable:

(*a*) To replace the old curve with a new and sharper curve located so as not to vary far from the old alignment.

(*b*) To replace a part of the existing curve with a curve of slightly smaller radius, compounding with the old curve.

(*c*) To make a new alignment for the main part of the curve close to the old and replace a part of this with a curve of smaller radius.

81. To replace the entire curve. First method.—In Fig. 6, the dotted line TNH is the old curve, T being its P.C. It is desired to throw the line out at H, the middle point of the curve, a distance of HK $= p$, and replace the curve by a sharper curve whose P.C. will be at D, thus permitting the spiral AEL to be inserted. P is the intersection of tangents, which comes outside the diagram. Let R_1 be the radius of the old curve and R of the new. HP — KP $= p$, or

$$R_1 \text{ exsec } \tfrac{1}{2} I - (R + o) \text{ exsec } \tfrac{1}{2} I - o = p.$$

ence

$$R_1 - R = o + \frac{o+p}{\text{exsec } \frac{1}{2}I} = \frac{o+p}{\text{vers } \frac{1}{2} I} - p \,\ldots\ldots\ldots\, (24)$$

$$p = \frac{(R_1 - R) \text{ vers } \frac{1}{2} I - o}{\cos \frac{1}{2} I}$$

$$= (R_1 - R - o) \text{ exsec } \frac{1}{2} I - o \,..\, (25)$$

Also $AT = AP - TP = t - (R_1 - R - o) \tan \frac{1}{2} I$

$$= t - (o+p) \cot \tfrac{1}{4} I \,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\, (26)$$

y which the P.S. (A) may be located; or if T is not nown, the tangent distance AP may be calculated and A cated.

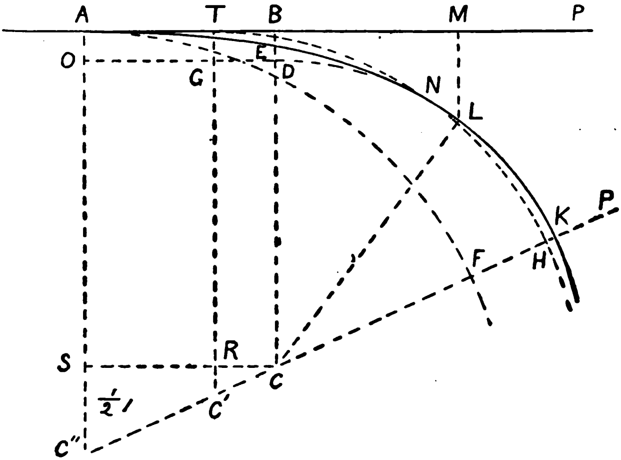

FIG. 6.

82. Values of p from zero to $\frac{1}{2}o$ may be used. If ie new curve comes inside the old at the center, p must e used as negative and its sign in the formula must be hanged. It must be borne in mind that the o used in the bove formula must be the o of the new curve. As this ill not be known, first use the value of o for the old

curve in (22), select a radius and degree of new curve near the resulting value, and then determine p and AT with the o for the new curve.

83. As an example take $I = 60°$, $D = 6°$, $a = 2$. Then o for a 6° curve is 3.93. Take 1.0 ft. as a trial value of p. By equation (24) the radius of the new curve will be approximately 35.8 ft. shorter than the old and by consulting a table of radii of curves it will be seen that a 6° 14′ curve may be used. $\frac{D_1}{a} = 3.117$; there will be 311.7 ft. of spiral at the end.. The o for a 6° 14′ curve will be 4.4 ft. and the resulting p is found by (25) to be 0.5 ft. There will be 9° 43′ in each of the spirals and 40° 34′ in the remaining circular curve. The P.S. may be located by measuring the tangent-distance T, or the middle point K of the curve may be located by means of the external distance, E.

84. Second method. The method just described may be modified to use measurements along the external secant as follows, using Fig. 6 as before: Intersect tangents at P. (Intersection outside of diagram.) Measure PK along external secant to the point K where it is desired to have middle point of new curve come. By equation (20) (page 15) calculate the radius and the degree-of-curve which will give PK as the external-distance E of a spiraled curve. It will be necessary to use the value of o for a degree-of-curve equal to that of the original curve, since the degree of the new curve is not yet known. Next select a curve whose degree will give a radius close to that found by the above calculation. For this D, compute o, and also PK. As the real o was not known in the first calculation and the new curve will not have exactly the R found, the point K as now located may not coincide with that first chosen. Having located K anew, the curve

may be run in from K with back-sight on P, or the tangent-distances may be measured to locate P.S.

Instead of using equation (20), PK may be found by adding o sec $\frac{1}{2}$ I to the external-distance for I° of circular curve without spiral. Likewise in finding the desired D, subtract o sec $\frac{1}{2}$ I from the measured distance PK, and use the remainder as the external-distance for an unspiraled circular curve. By this means a table of external-distances for a 1° curve may be utilized and the calculations shortened.

85. This method is applicable on short curves and where the ground will permit of easy and accurate measurement of the external-distance.

Take the same example as before. Consider that the measured PK is 146.7 ft. Using $o = 3.93$, o sec $\frac{1}{2}$ $I = 4.5$ ft. The circular curve whose external-distance is $146.7 - 4.5 = 142.2$ ft. lies between 6° 14′ and 6° 13′. Choosing a 6° 14′ curve and recalculating, the external-distance for a simple curve is found to be 142.2 and o sec $\frac{1}{2}$ I 5.1, making PK 147.3 ft. After K is located the curve may be run in.

86. (b) To replace a part of the curve. In Fig. 7, B is the P.C. of the old curve whose degree is D_0. It is desired to go back on this curve a distance BD and there compound with a curve of somewhat sharper curvature, D_1, which if run to a point E where its tangent is parallel to the original tangent shall be at a distance EF $= o$ from it. The tangent and D_1 curve may then be connected by a spiral having this o. It is required to locate D and the P.S. and P.C.C. so that a selected curve, D_1, will give a calculated or assumed distance EF as o.

Let R_0 be the radius of the D_0 curve and R_1 that of the D_1 curve, and I_1 the angle to be replaced.

$o = \text{EF} = \text{FH} - \text{EH} = (R_0 - R_1) \text{ vers } I_1.$

$$\text{vers } I_1 = \frac{o}{R_0 - R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Having I_1, back up on the curve to D, run the D_1 curve to G, the P.C.C. of spiral, and locate the spiral.

The P.S. may be located from B by

$$\begin{aligned} AB &= t - (R_0 - R_1 - o) \ \tan I_1 \\ &= t - (R_0 - R_1) \ \sin I_1 \ldots\ldots\ldots\ldots\ldots\ldots (28) \end{aligned}$$

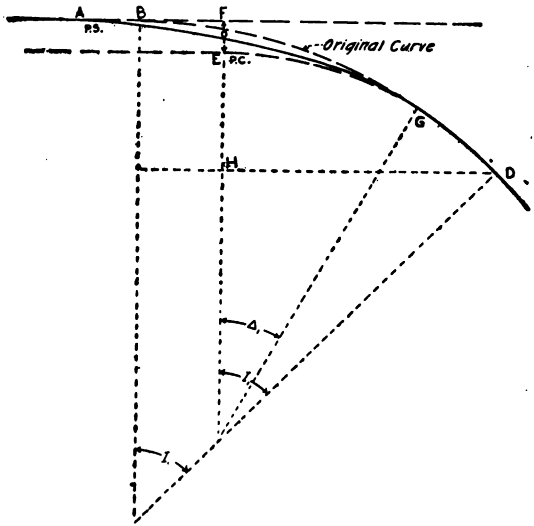

FIG. 7.

87. Thus, consider that a part of a 4° curve is to be replaced with a 4°30′ curve, and that $a = 1$. $o = 6.62$. By equation (27), $I_1 = 16°\ 35'$. Take out BD = 414.6 ft. of 4° curve and locate D. 16° 35′ of 4° 30′ curve requires 368.5 ft. The half length of the spiral is 225 ft. The P.C.C. is then found by running from D 368.5 — 225 = 143.5 ft. of 4° 30′ curve to the P.P.C., G. Likewise by (28) A.B. = 224.8 — 45.4 = 179.4 ft.

88. The limiting values of D_1 will be on the one hand $\frac{4}{3}$ D_0 and on the other a value which will make BD one half of the length of the original curve. Ordinarily, D_1 should not be one fifth more than D_0; better less than one tenth more on sharp curves.

89. It may be convenient to calculate a standard set of values for the curves on a road. The following gives a few such values.

D_0	a	D_1	$R_0—R_1$	o	I_1	AB	GD
2°	½	2°15′	318.3	3.31	8°16′	179.2	142.4
2°	½	2°30′	572.9	4.53	7°12′	178.1	38.0
3°	1	3°30′	272.8	3.12	8°40′	133.8	72.6
4°	1	5°	286.4	9.07	14°27′	178.1	39.0
5°	1	6°	190.9	15.65	23°21′	223.4	89.2
5°	2	6°	190.9	3.91	11°38′	111.4	43.9

90. (c) To re-align and compound. When the middle portion of the curve is in fair alignment and it is desired not to disturb it, or when it seems best to re-locate the central part of the curve, a method by taking up points on the old track and not running the principal tangents to an intersection, may be used. See Fig. 8. Select M, N, and O on the curve on the portion not to be disturbed. Set transit at M, measure the distances MN and NO, and by the usual methods for circular curves determine the degree of curve, D_0, which will fit this middle portion. Or select points that will locate the curve in a desirable position, and determine D_0. The selection of points in this way will probably not give a curve whose tangent coincides with the track tangent. When this curve is run back until its tangent is parallel to AH at B, the distance from the track tangent will be called m. From M, intersect with tangent at H and measure I. Determine m by running out MDB and measuring the

offset or by calculation from $m = HG \sin I$ and HG=HM — GM, remembering that GM is the tangent-distance for $I°$ of D_0 curve. Let ED be the new D_1 curve which must be run in from D so that EF shall be the o for the D_1 curve. Call the radius of the D_0 curve R_0 and that of the D_1 curve R_1. The D_1 curve will have I_1 of central angle. Then

$$EK = o - m = (R_0 - R_1) \text{ vers } I_1.$$

$$\text{vers } I_1 = \frac{o - m}{R_0 - R_1} \quad \text{.........................} (29)$$

If o is less than m, then R_1 must be greater than R_0.

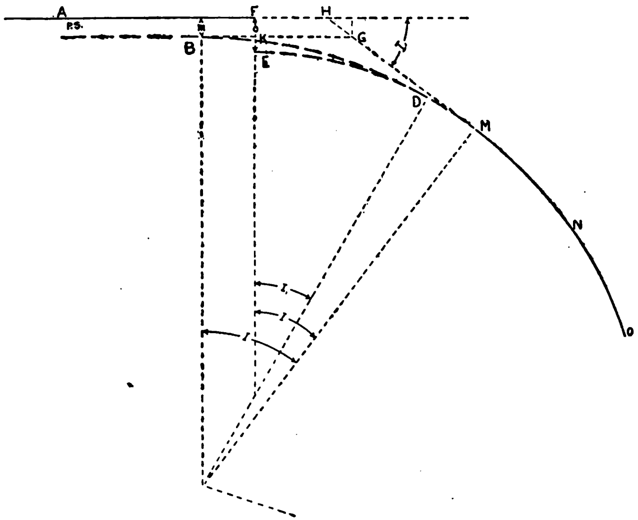

FIG. 8.

If K comes outside of AH, m must be added to o. Care must be taken that M is far enough back on the curve.

91. To locate the curve, run $I - I_1$ of D_0 curve from M to D. Run DE to locate the P.C., or run such part of this D_1 curve as will give the P.C.C. for the spiral.

The P.S. (A) may be located as follows:

$$AH = t + BG - BK - HG \cos I \ldots\ldots\ldots\ldots\ldots (30)$$

$BG = GM$, the tangent-distance for $I°$ of D_0 curve, $BK = (R_0 - R_1) \sin I_1$, and HG cos I is also m cot I. The P.S. may also be located by offsetting o from E to F and measuring t to A.

92. For example, if D_0 has been found to be 4° and I at H 20° and m 1.2 ft., select $D_1 = 4° 30'$ and $a = 1$. Then $c = 6.62$. From equation (29) $I_1 = 15°$ Then calculating the length of the curves from the angles I and I_1, as MB is 500 ft. and DB 375 ft., MD = 125 ft. DE = 333.3 ft. since there is to be 15° of 4° 30′ curve. The P.C.C. for spiral is 333.3 — 225 = 108.3 ft. from D toward E, since the half length of the spiral is 225 ft. AH is 224.8 + 253.6 — 41.1 — 3.3 = 434.0 ft. The spiral may be run in by usual methods.

93. The limiting values of D_1 are similar to those given in the preceding method. Generally D_1 may be from one tenth to one fourth more than D_0, depending upon the amount of the curve and its degree.

94. When the new D_1 curve is so much sharper that it is desired to connect it with the old by a spiral, the following method is applicable. Call o_1 the offset to tangent, and o_0 the offset between the two curves, the latter to be found as for compound curves. Then by a method similar to the foregoing,

$$\text{vers } I_1 = \frac{o_1 - o_0 - m}{R_0 - R_1 - o_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

95. Methods of track men. When curves are left without transition curves, many track men "ease" the curve by throwing the P.C. inward a short distance and gradually approaching the tangent a few rail lengths

away, while the main curve is reached finally by sharpening the curve for a short distance.

96. Another simple method for track which is aligned to a circular curve, consists in utilizing one of the properties of the transition spiral. In Fig. 1, page 5, let ABK be the original track line, B being the P.C. Select a length of spiral and calculate *o*, or select *o* and calculate the length, by a preceding method. At a distance from B equal to half the length of spiral (point of the curve opposite L) throw the track inward to L a distance equal to *o*. At B, the old P.C., throw the track to E, a distance half as great. Measure back from B half the length of the spiral to A for the beginning of the easement. Between A and L, line the track by eye, or calculate offsets from Table IX. The remainder of the main curve must then be thrown in the same distance as at L.

97. On long curves the latter work would be objectionable. It may be avoided by using a spiral running up to a curve whose degree-of-curve is one third greater than that of the main curve and compounding directly with the main curve. To do this, first select length of spiral for a curve one third sharper than the circular curve which call *L*. See Fig. 9. Call the circular curve D_0 and the curvature of the end of the spiral D_1. Measure back from the old P.C. on tangent a distance $\frac{1}{3}$ *L*, which will locate the P.S. Measure forward on the curve from the P.C. a distance $\frac{1}{3}$ *L* to locate the middle of the spiral, and offset from prolongation of tangent a distance equal to $\frac{1}{2}$ *o*, or $\frac{1}{4}$ *o* from the circular curve. Measure also along the curve from the P.C. a distance $\frac{2}{3}$ *L* to the P.C.C. where the track will not be changed. The spiral will pass the old P.C. at $\frac{4}{27}o$ from it, and at a point $\frac{1}{3}$ *L* from the P.C.C. will be $\frac{1}{8}o$ distant from the circular curve.

The spiral is one and one half times as long as the circular curve replaced.

The o used must be that for the full spiral and for the sharper curve, $\frac{4}{3}$ D_0, and the true position of the circular curve should be known. As the last fourth of this spiral is sharper than the main curve, the elevation of the other rail up to the P.C.C. must be greater than that on the main curve, gradually reducing beyond to the regular amount.

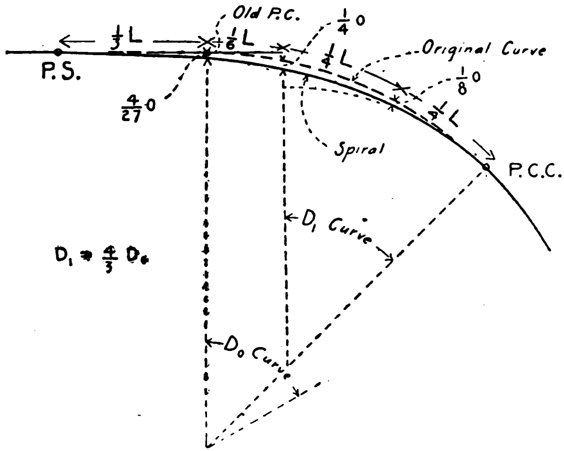

FIG. 9.

98. Thus, for a 3° curve using $a = 1$, the degree at the end of the spiral will be $3 \times \frac{4}{3} = 4$, and the length of spiral required is 400 ft., $o = 4.65$. The P.S. will be 133.3 ft. back of the P.C., the middle of spiral 66.7 ft. ahead of P.C. and the P.C.C. 266.7 ft. ahead of P.C. At the P.C. the track must be thrown in 0.69 ft., at the middle point 2.32 ft. from tangent (1.16 ft. from curve), and at the third quarter point .58 ft. from curve, while at the

P.C.C. there will be no change. Between these points the track may be aligned by eye, or ordinates may be calculated by Table XII.

However, while such methods are easements, they are at best makeshifts and should give place to better methods.

COMPOUND CURVES.

99. The spiral may be used to connect curves of different radii, choosing that part of the spiral having curvature intermediate between the degrees of the two curves, thus, connect a 3° and an 8° curve by omitting the spiral up to $D=3°$ and continuing until $D=8°$. In Fig. 10, DKM is a D_1 curve, and LNP a D_2 curve, the two curves having parallel tangents at M and N. D_2 is greater than D_1. Call the distance MN o. It is desired to connect the two curves by a spiral shown by the full line KP. The degree of curve of the spiral at K must be D_1 and at P, D_2. Consider the spiral to be run backward from K to a tangent at A. Then the spiral from K to P is the portion of the regular spiral from where its degree is D_1 to the point where it is D_2. Since the spiral diverges from the osculating circle at the same rate as from the tangent at the P.S., PN = MK and the spiral bisects MN. MN, or o, is the offset for a spiral for a curve whose degree is $D_2 - D_1$. Hence, find o for a $D_2 - D_1$ curve, and make the offset at MN. Measure MK and NP each equal to $\frac{1}{2}\,\frac{D_2 - D_1}{a}$ thus locating the P.C.C. of each curve K and P. Run in the spiral from K or P by the method for point on spiral heretofore described, AK being omitted. The angle between tangents at K and

P is Δ for a D_2 spiral minus Δ for D_1 spiral, and may also be expressed as $\frac{1}{2}(D_1+D_2)$ times KP in stations. Thus, with $a=2$, to connect a 3° and an 8° curve $o=2.27$, the value for a 5° spiral. The portion of the spiral used will be 250 ft. long. K is 125 ft. from M. and N is 125 ft. from P. If greater accuracy is required, the *t* Cor. for

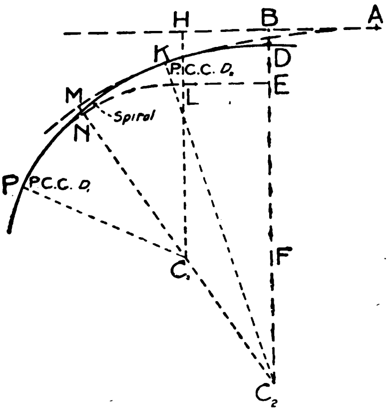

Fig. 10.

this length should be subtracted from 125. The angle between tangents at K and P is $\frac{1}{2}\times 2.50\,(3°+8°)=$ 13° 45′.

The spiral may also be used to connect two curves having a given offset between them.

100. To insert in old track. It may be desired to insert a spiral between the two curves of an existing compound curve by first replacing a part of the sharper curve with a curve of slightly smaller radius.

In Fig. 11, let AB be a D_1 curve and BG a D_3 curve, B being the P.C.C. and the D_3 curve having the smaller radius. C_1 is the center of the D_1 curve, not on the cut.

It is desired to go back on the D_3 curve to a point D and there compound with a D_2 curve which shall be run to a point E where its tangent shall have the same direction as the tangent to the D_1 curve produced backward to F has at F. The radial distance EF corresponds to the offset of the usual spiral and will be called o. It is desired to locate D and F so that a selected cuve, D_2, will give a calculated or assumed distance EF as o.

The distance EF is made up of FK and KE, the first being the divergence of the D_1 curve from the D_3 curve in

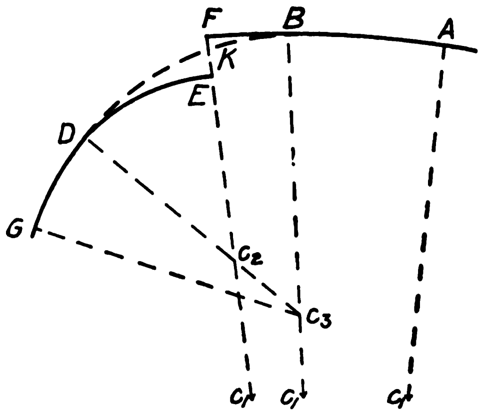

FIG. 11.

the distance BF and the second the divergence of the D_2 curve from the D_3 in the distance DE. Call the distance BF L_1, and DE L_2. For the small angles used these divergences may be calculated accurately enough by the approximate formula for tangent offset, $y = .87\ DL^2$, and we shall have

$$EF = .87\ (D_2 - D_3)\ L_2^2 + .87\ (D_3 - D_1)\ L_1^2 = o, \text{ or}$$

$$(D_2 - D_3)\ L_2^2 + (D_3 - D_1)\ L_1^2 = 1.15\ o \ldots\ldots\ldots (32)$$

Since the amount of D_1 curve in BF plus the amount

of D_2 curve in DE (total angle) must be equal to the amount of D_3 curve taken out, we have

$$D_2 L_2 + D_1 L_1 = D_3 (L_2 + L_1) \text{ or}$$

$$(D_2 - D_3)\ L_2 = (D_3 - D_1)\ L_1 \ldots\ldots\ldots\ldots (33)$$

Combining (32) and (33) and solving,

$$L_1^2 = 1.15 \frac{(D_2 - D_3)\ o}{(D_3 - D_1)\ (D_2 - D_1)} \ldots\ldots\ldots\ldots (34)$$

$$L_2^2 = 1.15 \frac{(D_3 - D_1)\ o}{(D_2 - D_3)\ (D_2 - D_1)} \ldots\ldots\ldots\ldots (35)$$

101. Having L_1 and L_2, the points D, E and F may be located, and the D_2 curve may be run in from D as far as necessary. The problem is then identical with that of putting a spiral between two curves having an offset o (EF) between their parallel tangents.

By the principles governing the placing of a spiral between two curves, it is seen that the length of the connecting spiral L' is that of a spiral for a curve of degree equal to the difference of degree of the two connected; that is

$$L' = \frac{D_2 - D_1}{a}$$

The offset is equal to that for a $(D_2 - D_1)$ degree curve from a tangent or

$$o = .0727\ (D_2 - D_1)\ L'^2 = .0727\ aL'^3 \ldots\ldots\ldots\ldots (36)$$

Half of this spiral will lie on one side of the offset and half on the other, hence in Fig. 12 $\frac{1}{2}L$ to the right of F will give the beginning of the spiral, H, and $\frac{1}{2}L$ to the left of E will give the end of spiral, I.

102. The method of field work will then be as follows: Measure from B, the P.C.C., (Fig. 12) back on the D_1 curve a distance BH $= \frac{1}{2} L' - L_1$ to locate the point of spiral H. Measure from B on the D_3 curve the distance BD $= L_1 + L_2$ to D, the new P.C.C., run in the D_2 curve to I, DI being $L_2 - \frac{1}{2} L'$. The spiral is then to be run in

from H to I. The dotted line in Fig. 12 shows the spiral.

The field work for the spiral is simple. The spiral may be run in by offsetting from the D_1 curve HF (Fig. 12) knowing that the offset from the curve to the spiral is the same as that of a spiral from the tangent using the distance from H as the distance on the spiral. Likewise the remainder of the spiral may be offsetted from the D_2 curve IE using distances from I in the calculations.

If the field work on the spiral is to be done by deflec-

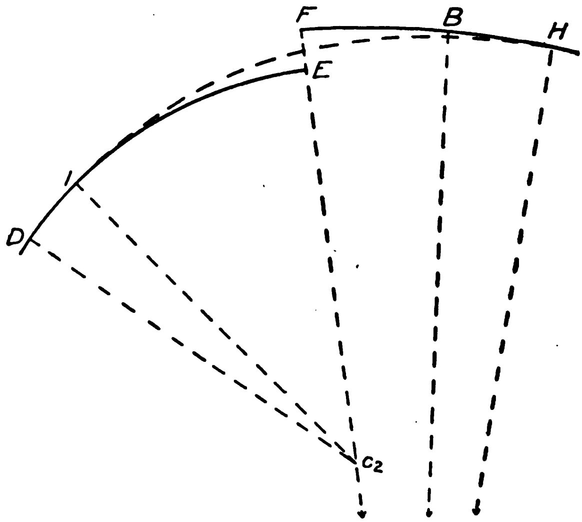

FIG. 12.

tion angles, the spiral may be run in from H by using as deflection angles the sum of the deflection angle for the circular curve HF and the spiral deflection angle from a tangent for the same distance; or the transition spiral may be run backward from I in a similar manner. In either case the work will be no more difficult than for spirals for simple curves.

103. As an example let us consider that a 2° and an 8° curve are compounded at B. Consider that the degree of the new curve to be run in is 8° 30′, and that the value of a to be used is 2. Then $D_1 = 2$, $D_3 = 8$, $D_2 = 8\frac{1}{2}$. For a spiral from 2° to 8° 30′, the value of the offset o (EF) is the same as the o for a 6° 30′ curve from a tangent. Hence $o = 4.99$. By formula (34), $L_1 = .271$, and by formula (35), $L_2 = 3.255$. Hence the point D will be back on the D_3 curve 325.5 + 27.1 or 352.6 ft. from B. The length of the spiral to be used will be

$$L' = \frac{8\frac{1}{2} - 2}{2} = 3.25.$$

Of this 162.5 ft. will be to the left of E and 162.5 ft. will be to the right of F. Hence H and I, the ends of the spiral, may readily be located and the spiral may be run in.

104. By this method the value of a may be chosen beforehand, the value of o may be easily calculated, and the preliminary field work is small. It may be stated that the limiting values of D_2 will be, on the one hand, a value so near D_3 that the resulting L_2 will carry the new point of compound curve back to the end of the old curve, and on the other hand such that the length of the D_2 curve shall be at least equal to half the length of the transition spiral, a value which may be shown to be $D_2 = \frac{1}{3}(4D_3 - D_1)$.

For large angles the above method is subject to slight error.

MISCELLANEOUS PROBLEMS

105. To change tangent between curves of opposite direction. Having given two curves of opposite direction connected by a short tangent, it is required to find the position of a line to which both curves may be connected by spirals. This involves determining the angle which must be added to each curve to get the position of the new P.C. of each curve for spiraling.

106. In Fig. 13 let AB be the original tangent connecting the two curves and l its length in feet. It is required to run the curve KA to E, which will be the

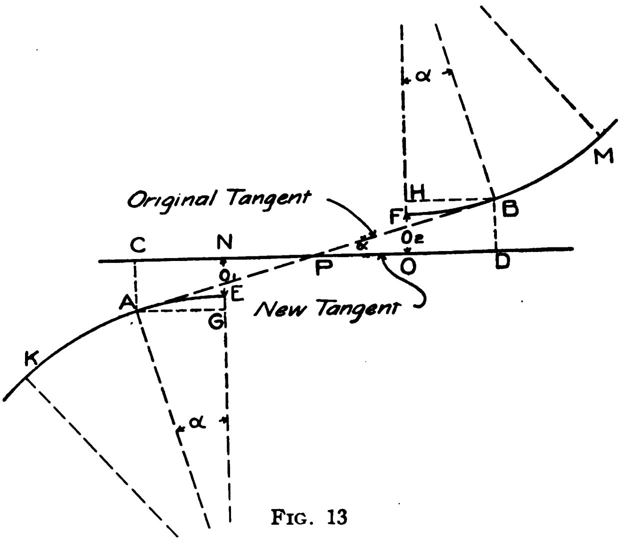

FIG. 13

P.C. for the spiraled curve, and MB to F for its spiraled P.C., and also to find the position of the line CD, which will be the common tangent for the two spirals. Call the

angle APC a. It is the same as that of the additional amount of curve AE and BF. Let R_1 be the radius of the curve KE and R_2 that of MF, and o_1 and o_2 be the respective spiral offsets NE and OF for the spirals chosen ior the two curves. $AC + BD = NE + EG + OF + FH$. Then, since $AC + BD = l \sin a$ and $EG = R_1$ vers a, etc.,

$$l \sin a = o_1 + o_2 + (R_1 + R_2) \text{vers } a \quad \text{.............. (37)}$$

107. Since a will be small, we may substitute, using a in degrees, $\sin a = .01745\ a$ and vers $a = .000152 a^2$ which are close approximations below 8°. Transforming,

$$a = \frac{o_1 + o_2}{.01745\ l} + \frac{.000152\ (R_1 + R_2)\ a^2}{.01745\ l}$$

This quadratic may be solved, but usually the following approximate root gives sufficiently close results:

$$a \text{ (in degrees)} = \frac{o_1 + o_2}{.01745\ l} + \frac{.000152\ (R_1 + R_2)\ (o_1 + o_2)^2}{(.01745\ l)^3} \quad (38)$$

Having a the lengths AE and BF may be found, the position of the P.C.C. of each curve found, and the new tangent located by offsetting EN and FO, or by offsetting AC (equal to $o_1 + R_1$ vers a) and BD. For very short tangents, spirals must be chosen short enough not to overlap on the tangent.

108. As an example take a 3° curve and a 4° curve connected by 600 ft. of tangent. Use $a = 1$. Then

$o_1 = 1.96$ and $o_2 = 4.65$. By equation (38)

$a = .63 + .02 = .65° = 0°\ 39'$.

This result checks equation (37) very closely. 0°39′ gives 21.7 ft. of 3° curve (AE) and 16.2 ft. of 4° curve (BF). There will be 300 ft. of spiral for the 3° curve and 400 ft. of spiral for the 4° curve. The P.S. and P.C.C. of each spiral will be half of the spiral length from the points E and F. The P.C.C. of one will be (150 —21.7=128.3) ft. back of A, and of the other (200—16.2

=183.8) ft. back of B. AC will be $(1.96+.11=2.07)$ ft. and BD $(4.65+.09=4.74)$ ft. The distance from C to P.S. will be $150+21.7$, and from D to the other P.S, $200+16.2$, neglecting the t correction. The spirals may then be run in as usual.

109. This solution may also be applied to the case where a tangent thrown off from the curve KA does not strike the curve MB but is parallel to this curve at a point opposite B distant m from it. Since $\cos a$ is nearly 1, equations (37) and (38) may be modified by subtracting m from (o_1+o_2) wherever it occurs. This modification is of convenience in revising old lines. The engineer should make his own diagram.

110. To change tangent between curves of same direction. Having given two curves of same direction connected by a tangent it is desired to find the position of a line to which the two curves may be connected by spirals. As in the preceding problem this involves determining the change in the angle of the two curves and the position of the P.C. of each curve for spiraling.

111. In Fig. 14 let AB be the original tangent connecting the two curves and l its length in feet. It is required to back up on the curve AK to E for the P.C. for spiraled curve and to run the curve MB to F for its spiraled P.C., and to find the position of the line CO which will be the common tangent for the two spirals. Call the angle BAD′ a. It is the same as that in AE and BF. Let R_1 be the radius of the curve KE and R_2 that of MF, and o_1 and o_2 be the respective spiral offsets NE and OF for the spirals chosen for the two curves.

$$BD - AC = OF + FH - NE - EG.$$

Then, since BD—AC or BD′ equals $l \sin a$ and EG equals R_1 vers a, etc.

$$l \sin a = o_2 - o_1 - (R_1 - R_2) \text{ vers } a \ldots\ldots\ldots\ldots (39)$$

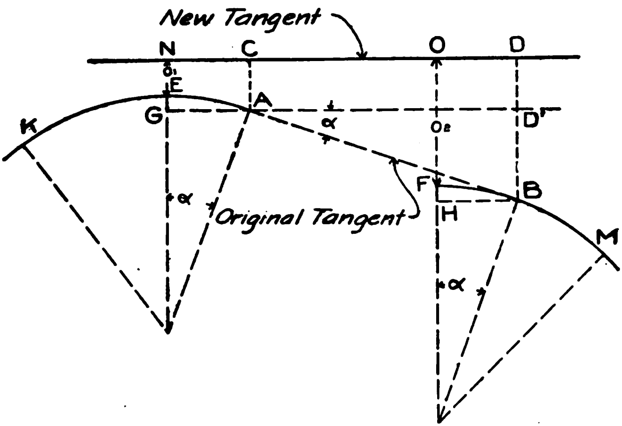

FIG. 14.

112. Since a will be even smaller than in the preceding problem, we may substitute $\sin a = .01745_a$ and vers $a = .000152a^2$, using a in degrees. Transforming,

$$a = \frac{o_2 - o_1}{.01745\ l} - \frac{.000152\ (R_1 - R_2)}{.01745\ l}\ a^2$$

As in the preceding problem, the approximate solution of this quadratic may be used.

$$a \text{ (in degrees)} = \frac{o_2 - o_1}{.01745\ l} - \frac{.000152\ (R_1 - R_2)\ (o_2 - o_1)^2}{(.01745\ l)^3} \quad (40)$$

The last term here is very small.

Having a the lengths AE and BF may be found, the position of the P.C.C. of each curve found, and the new tangent located by offsetting EN and FO, or by offsetting AC (equal to $o_1 + R_1$ vers a) and BD (equal to $o_2 + R_2$ vers a. The length of spiral must not be so great that the spirals will overlap on short tangents.

113. As an example take a 3° curve and a 4° curve connected by 600 ft. of tangent. Use $a = 1$. Then $o_1 = 1.96$ and $o_2 = 4.65$. By equation (40) $a = .257 - .006 = .251 = 0° 15\frac{1}{2}'$.

$0° 15\frac{1}{2}'$ gives 8.6 ft. of 3° curve (AE) and 6.4 ft. of 4° curve (BF). There will be 300 ft. of spiral for the 3° curve and 400 ft. for the 4° curve. The P.C.C. of spiral will then be $(150 + 8.6 = 158.6)$ ft. back of A and $(200 - 6.4 = 193.6)$ ft. back of B. AC will be 1.98 ft. and BD 4.67 ft. The distance from C to P.S will be $150 - 8.6$ and from D to the other P.S. $200 + 6.4$, neglecting the t correction. The spirals may then be run in.

114. This solution may also be applied to the case where a tangent thrown off from the curve KA misses the curve MB by a distance m from B, the point of parallelism. In this case equations (39) and (40) may be modified by subtracting m from $(o_2 - o_1)$ wherever it occurs. If the second curve is one of larger radius, it will be necessary to construct a new diagram and determine the signs of the terms.

UNIFORM CHORD LENGTH METHOD

115. The treatment of the spiral heretofore given is based upon principles which permit the use of any chord length, either uniform or variable, throughout the length of the spiral. Regular chord lengths, like 20 or 25 feet, may be used, if desired and the excess if any used as a fractional chord at the beginning or the end of the spiral. If it is desired to use chords of common length, another method known as uniform chord length method, may be derived by modifying the preceding formulas. A further modification of this method may be made to allow the

use of fractional chord lengths at the beginning or the end of the spiral, so that it will not be necessary to make the uniform chord length an aliquot part of the length of the spiral. Thus, if the spiral is to be 203.2 ft. long, ten 20-ft. chords, or eight 25-ft. chords, or thirteen 15-ft. chords, etc., may be used—the first or last chord or both being fractional.

116. The notation used will be the same as heretofore except as noted, and the equations will be numbered the same, using the prime mark to distinguish them. Let c be the chord length used. This will be expressed in hundreds of feet, that is in the number of 100-ft. stations.

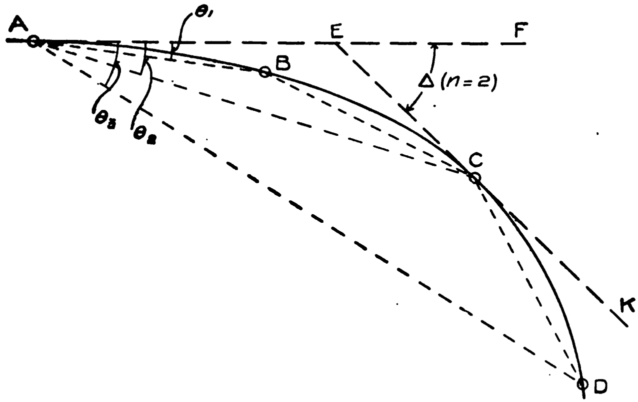

FIG. 15.

For a chord length of 20 ft., $c = .2$; for one of 15 ft., $c =$ .15, etc. Let n (an integer) represent the number of full chords from the P.S. to a desired point. In Fig. 15, A is the P.S. and its n is 0. The n of B is 1, of C 2, etc. Let $\theta_1 =$ spiral deflection angle at P.S. from initial tangent for a single full chord length (BAE) (called unit spiral deflection angle), and θ^n for n chord lengths. For $Dn =$

3 and $\theta_n = \text{DAE}$. Similarly, Δ_n, D_n and L_n are for a point n chord lengths from the P.S. For the instrument at other points than P.S., let n' be the number of chord lengths from the P.S. to the chord point at which the instrument is located, reserving n still as the number of chord lengths from the P.S. to the point to be located, and let Φ_n represent the deflection angle from tangent at the instrument point to this desired point. Thus, if the instrument is at C two chord lengths from the P.S. $n' = 2$, and to locate D three chord lengths from the P.S., $n = 3$ and Φ_n will represent the deflection angle DCK to locate the third chord point D.

117. The length $L = nc$. By substitution in equations (1), (2), and (9) of the spiral, we have for any point on the spiral distant $L = nc$ from the P.S.

$$D = a\ L = a\ n\ c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1')$$

$$\Delta = \tfrac{1}{2}\ a\ L^2 = \tfrac{1}{2}\ a\ n^2\ c^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2')$$

For end of first full chord and for end of n full chords, respectively,

$$\left.\begin{array}{l} \theta_1 = \frac{1}{6}\ a\ c^2 = \frac{1}{3}\dfrac{c^2}{L_n}\ \Delta_n = \frac{1}{6}\dfrac{D_n\ c^2}{L_n} \\ \theta_n = \frac{1}{6}aL^2 = \frac{1}{6}a(nc)^2 = n^2\theta_1, \end{array}\right\} \ldots\ldots\ldots\ldots (9')$$

Also $\Phi_n = \frac{1}{2}aL'(L - L') \pm \frac{1}{6}a(L - L')^2$

$$= \tfrac{1}{2}\ a\ n'\ c\ (n - n')\ c \pm \tfrac{1}{6}\ a\ (n - n')^2 c^2.$$

$$= [3n'\ (n - n') \pm (n - n')^2]\ \theta_1 \ldots\ldots\ldots\ldots (10')$$

In formula (10′), the arithmetical difference of the numbers of the chord points is taken, rather than the algebraic difference. If the latter is used, the signs of operation should all be plus.

118. The first step is to calculate the value of the unit spiral deflection angle θ_1 by means of equation (9′), using a and c or other terms. For a chord length of 20 ft., and a value of $a = 2$, $c = .2$ and $\theta_1 = \frac{1}{6} \times 2 \times (2)^2 =$

0° 0.8′. If a spiral 250 ft. long is to connect with a 4° curve using 25-ft. chords, $D_n = 4$ and $L_n = 2.5$, $\theta_1 = \frac{1}{6} + 4 \times \frac{1}{16} \times \frac{1}{2.5} = 0° \, 1'$. If $\Delta_n = 9°$, $L = 3$ and $c = .2$ (20 ft.), $\theta_1 = \frac{1}{3} \left(\frac{2}{3}\right)^2 \times 9° = 0.8'$.

119. The value of θ_1 gives a basis for computing the deflection angle for other points; thus for a point 5 chord lengths from the P.S., $n = 5$ and the deflection angle by equation (9′) is 25 times the value of θ_1. For the instrument at the fifth chord point ($n' = 5$), the deflection angle from the tangent at the instrument point to a point 8 chord lengths from the P.S. ($n = 8$) is by equation (10′): $\Phi_n = [3 \times 5 \ \ (8 - 5) \pm (8 - 5)^2] \ \theta_1 = 54 \ \theta_1$. To locate from the same instrument point a point 3 chord lengths from the P.S. (2 from the instrument point), the deflection angle is $26 \ \theta_1$.

120. Table of unit spiral deflection angles. A table giving θ_1 for various chord lengths for many of the values of a used in field work may be of service. Table XIII gives spiral deflection angles for first chord length (unit spiral deflection angles). The angle is given in minutes. It is well in the calculations to express decimals as common fractions; thus, for 20-ft. chords with $a = 1$ use $\theta_1 = .5\frac{1}{3}'$; for 16-ft. chords with $a = 1\frac{2}{3}$ use $\theta_1 = .42\frac{2}{3}'$.

121. Table of coefficients for deflection angles. The values obtained from (9′) and (10′) may be considered as coefficients of θ_1, and a general table prepared. Table XIV is table for coefficients for a spiral up to 15 chord lengths for use with the instrument at any chord point.

122. For the instrument at the P.S., multiply the coefficient in the columns headed 0 opposite the chord point to be located by the value of the spiral deflection angle for a single chord length (unit spiral deflection angle θ_1).

123. To find the deflection angle from the tangent at any chord point, enter the column whose heading gives the number of the chord point at which the instrument is placed and take the coefficient opposite the number of the chord point to be located; then multiply the spiral deflection angle for a single chord length (θ_1) by this coefficient. Thus, as in example cited above, for the instrument at 5, the deflection angle from the tangent at this point to locate a point 8 chord lengths from the P.S. is found to be 54 θ_1, and to locate a point 3 chord lengths from the P.S. is 26 θ_1. This table may easily be extended. The variation in the tabular differences in horizontal, vertical, and diagonal directions is readily discerned, and if preferred the method of differences may be used for calculating deflection angles for a particular case in place of a multiplication of these coefficients.

124. To end the spiral with a fractional chord. If the number of chord lengths is not integral, the first and succeeding chords may be made of uniform length until the last one is reached and the full deflection angle may be turned off for the P.C.C. Thus, for 218.4 ft. of spiral, ten 20-ft. chords may be used and the full deflection angle turned off for the remaining 18.4 ft.

125. To begin the spiral with a fractional chord length. In case it is desired to begin the spiral with a fractional chord length, the following modification may be made. Let m be the ratio of this fractional chord length to a full chord length, and θ_m be the spiral deflection angle from the initial tangent for this fractional chord length, which from the general formula for θ may be seen to be $m^2 \cdot \theta_1$. Let θ_{n+m} be the spiral deflection angle from initial tangent to locate a point $(n+m)$ chord lengths away (n an integer and m fractional), and Φ_{n+m} the deflection angle from tangent at instrument point $(n'+m)$ chord lengths from

P.S. to locate a point $(n+m)$ chord lengths from P.S. Substituting in formula (9) page 8,

$$\theta_{n+m} = \tfrac{1}{8}\ a\ (n+m)^2\ c^2 = (n+m)^2\ \theta_1 = (n^2+2nm+m^2)\ \theta_1$$
$$= \theta_n + n\ \ (2m\ \ \theta_1) + \theta_m \ldots\ldots\ldots\ldots\ldots\ldots (9'')$$

126. In the last member of equation (9″), the first term is the spiral deflection angle for n full chords, the second term is n times a constant, and the third term is the spiral deflection angle for the fractional chord. The calculations may be simplified by the method of differences.

For example, for a chord length of 20 ft. let the beginning chord be 8.4 ft. Then $m = \frac{8.4}{20} = .42$. If 1.2′ be the unit spiral deflection angle θ_1, $\theta_m = .21'$. To locate a point 88.4 ft. from P.S., the spiral deflection angle at P.S. will be

$$\theta_{n+m} = 16 \times 1.2 + 4 \times 2 \times .42 \times .21 + .2 = 23'.4.$$

Use Table XIV in calculating θ_n.

127. For the instrument at a chord point $(n'+m)$ chord lengths from the P.S., the deflection angle from the tangent at this chord point to locate a chord point $(n+m)$ chord lengths from the P.S. is found from equation (10) page 11 to be

$$\Phi_{n+m} = [3\ (n'+m)\ \ (n-n') \pm (n-n')^2]\ \ \theta_1$$
$$= \Phi_n + (n-n')\ \ (3m\ \ \theta_1) \ldots\ldots\ldots\ldots\ldots\ldots (10'')$$

In the last member of equation (10″) the first term is the deflection angle for full chords, and the second term is $(n-n')$ times a constant. If n' is greater than n, the second term is still added numerically to the first. Tabular differences may also be used.

128. To use equation (10″) first calculate Φ_n using Table XIV; then add the last term. Thus, for a chord length of 20 ft. and a beginning chord of 8.4 ft. and a

unit spiral deflection angle of 1.2′, with the instrument 88.4 ft. from P.S., $n' + m = 4.42$. To locate a chord point 148.4 ft. from P.S. ($n = 7$), the deflection angle from the tangent is found to be, taking the coefficient for Φ_n from the column headed 4,

$$\Phi_{n+m} = 45 \times 1.2' + 3 \times 3 \times .42 \times .21' = 54.8'.$$

To locate a chord point 48.8 ft. from P.S., using the same instrument point, ($n = 2$, $n' = 4$, $m = .42$), we have for the deflection angle

$$\Phi_{n+m} = 20 \times 1.2 + 2 \times 3 \times .42 \times .21' = 24.5'.$$

129. To illustrate the use of these methods, take the following examples. It is desired to spiral a 6° 40′ curve with 200 ft. of spiral, using 20-ft. chords. $c = .20$. $n = 10$. $D_n = 6\frac{2}{3}$. By equation (9′) the spiral deflection angle θ_1 for a 20-ft. chord is $\frac{1}{6} \times \frac{.20^2}{2} \times 6° 40' = 1\frac{1}{3}' = \theta_1$. The multiplication of $1\frac{1}{3}'$ by the coefficients in Table XIV for instrument at 0 and for instrument at 10, gives the desired deflection angles.

If it is desired to connect a tangent with a 4° curve so that the offset o shall be 5.0 ft., proceed as follows. By equation (14) the length of spiral is 415.2 ft. Using 25-ft. chords, the deflection angle in minutes is by equation (9′) $\frac{10 \times 4 \times (\frac{1}{4})^2}{4.152} = .602 = \theta_1$. Table XIV will give the coefficients for multiplication, and the fractional chord may be left for the last measurement.

130. To show the use of fractional beginning and ending chords, consider that a spiral 138.4 ft. long is to connect with a 10° curve and that it is desired to use 15-ft. chords but that the first chord shall be 9.3 ft. $c = .15$. $m = \frac{9.3}{15} = .62$. By equation (9′), the spiral deflection angle for a 15-ft. chord θ_1 is found to be $1\frac{7}{8}'$, and for

the point 9.3 ft. from P.S. θ_m is $.62^2 \times 1\frac{5}{8} = .62'$. The table below gives values for field work, considering that P.S. is at Sta. 322 + 13.7. In the column headed "number of

Survey Station	No. of Chord Point	Central Angle	INSTRUMENT POINT				
			P.S.	0.62	1.62	4.62	5.62
322+13.7	P.S.	0	0	1.2′		1°09.4′	
+23	0.62	1.8′	0.6′	0		1°04.1′	
+38	1.62	12.8′	4.3′	4 6′	0	53.0′	
+53	2.62	33.5′	11.2′	12.5′		38.6′	
+68	3.62	1°03.9′	21.3′	23.7′		20.9′	
+83	4.62	1°44.1′	34.7′			0	
+98	5.62	2°34.0′	51.3′			24.1′	0
323+13	6.62	3°33.6′	1°11.2′				
+28	7.62		1°34.3′				
+43	8.62		2°00.7′				
+52.1	End	6°55.3′	2°18.4′				

chord point" the integer of the number is n and the fractional part is m. In the succeeding column headings for the instrument points, the integer is n'. Thus, to determine the deflection angle with the instrument at 0 to locate 323 + 43, by equation (9″) and Table XIV,

$$\theta_{n+m} = (64 \times 1\tfrac{5}{8}) + (8 \times 2.01) + .6 = 2^\circ\ 00.7'.$$

To determine the deflection with instrument at 322 + 83, ($n' = 4$), to locate 322 + 68, ($n = 3$), by equation (10″),

$$\Phi_{n+m} = (11 \times 1\tfrac{5}{8}) + (1 \times 3.02) = 20.9'.$$

131. To run in the spiral from the P.C.C., this method may likewise be used if the P.P.C. is at n or at $n + m$ chord lengths from the P.S. If it is not, that is, if both the first and last chord lengths are to be fractional, the points on the spiral as far as the next instrument point may be set by the principle that deflection angles will equal the difference between the deflection angle for a circular curve and the deflection angle for a spiral from

initial tangent, both for a distance equal to the distance to the desired point. After the next instrument point is reached, calculate deflection angles as though working from the P.S.

132. The method of uniform chord lengths is subject to the same correction for θ as is given on page 9. As it is not likely that this method will be used for large deflection angles, the error will usually be negligible. For Φ the correction needed is almost exactly that for the θ which enters into it; thus in equation (10′) make a correction which would be necessary for a θ equal to $(n - n')^2 \theta_1$. This is not often necessary.

133. It will be seen that the method of uniform chord lengths may have advantages where a chord of full length may be used at the beginning of the spiral, especially where the rate a is fractional, and that it is also applicable when the beginning chord is fractional. It is more especially applicable where evenly spaced points are wanted. Table XIV is a convenient table.

STREET RAILWAY SPIRALS

134. For use in connection with curves of short radii, as street railway curves, the formulas for the transition spiral may be modified with advantage. The variable radius R of the spiral may replace the degree-of-curve D. The product of the radius at any point by its distance from the P.S. will be shown to be constant for a given spiral, and this product may be used as the characteristic constant, taking the place of a. The offset o may be used as one fourth of the ordinate y of the terminal point of the spiral except for extreme lengths. Certain other

approximations may be made which are not always allowable with curves of large radius.

135. Theory.—The general notation will not be changed. Fig. 16 shows the co-ordinates x and y, spiral intersection angle Δ, spiral deflection angle θ, and spiral tangent-distances u and v for a point on the spiral, and

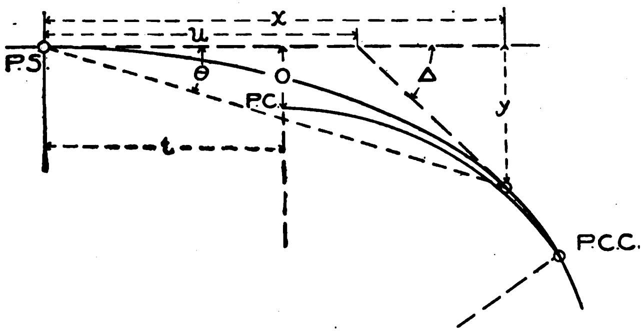

FIG. 16.

also o and t for the full spiral, together with the produced circular curve. As before, R = radius of curvature at any point and s = length of the spiral arc in feet from P.S. to any point of the spiral. From equation (1) page 6,

$$a = \frac{D}{L} = \frac{100D}{s} = \frac{573000}{s\ R}$$

Hence $$s\ R = \frac{573000}{a}, \text{ a constant.}$$

Represent this constant product of s and R by k. Then

$$k = s\ R = \frac{573000}{a} \text{ and } R = \frac{k}{s} \qquad (51)$$

The property of the transition spiral that R varies inversely as the distance along the spiral is satisfied by this equation.

136. Modification of the formulas already derived may be made as follows. The angle subtended by a circular arc in degrees is equal to $\frac{180}{\pi}\frac{s}{R}=\frac{57.3\,s}{R}$. Since Δ is one half as much as the angle of the same length of circular curve having a radius equal to the terminal radius of the spiral,

$$\Delta=\frac{28.65\ s}{R}=\frac{28.65\ s^2}{k} \qquad (52)$$

This may also be derived directly from equation (2) page 6 by substitution for the values of a and L. Also

$$\theta=\tfrac{1}{3}\,\Delta=\frac{9.55\ s}{R}=\frac{9.55\ s^2}{k} \qquad (53)$$

For large angles, if the precise values of θ are desired, the corrections given in the table on page 9 may be made. However, for the short distances involved this correction may generally be neglected.

137. Consider that $o=\frac{1}{4}\,y_1$, using the subscript $_1$ to designate the y, etc, of the terminal point of the spiral. By trigonometry $y_1-o=R_1$ vers Δ. Then $y_1=\frac{4}{3}\,R_1$ vers Δ and

$$o=\tfrac{1}{3}\ R_1 \text{ vers } \Delta \qquad (54)$$

Also by substitution for a and L in equations (6), (7), (13), (17), (21) and (23), pages 17 and 18, the following formulas are obtained:

$$y=\frac{s^3}{6k} \qquad (55)$$

$$x=s-\frac{s^5}{40\,k^2} \text{ or } s-\frac{9}{10}\frac{y^2}{s} \qquad (56)$$

$$o=\frac{s_1^{\,3}}{24\,k} \text{ or } \frac{s_1^{\,2}}{24\,R_1} \qquad (57)$$

$$t=\tfrac{1}{2}\,s_1-\frac{s_1^{\,5}}{240\,k^2} \qquad (58)$$

$$C = s - \frac{s^5}{90\,k^2} \dots\dots (59)$$

$$v = \tfrac{1}{3}\,s + \frac{s^5}{120\,k^2} \dots\dots (60)$$

In the last three equations, note that the t correction is $\frac{1}{6}$ of the x correction (last term in equation (56) used in finding x from s); the C correction is $\frac{4}{9}$ of the x correction; and the v correction is $\frac{1}{3}$ of the x correction.

For extreme cases, the values of y and o given by equations (55) and (57) will be slightly too large. For y, subtract $.003\frac{s^7}{k^3}$ from the results of equation (55). For o subtract one eighth as much. For x, t, C, and v, the terms given in the equations above will generally be sufficiently accurate.

138. The following equations may be repeated here:

$$T = t + (R_1 + o)\ \tan\ \tfrac{1}{2}\ I \dots\dots (18)$$

$$E = (R_1 + o)\ \text{exsec}\ \tfrac{1}{2}\ I + o \dots\dots (20)$$

$$\left.\begin{aligned} u &= x - y\ \cot\ \Delta \dots\dots \\ u &= x - v\ \cos\ \Delta \dots\dots \end{aligned}\right\} (22)$$

139. The Tables—Tables will facilitate the application of these equations. Tables XV-XIX give properties of five spirals. The spiral is to be used up to that length which gives the required radius. The x correction is the amount to be subtracted from the length of the spiral to give the abscissa x. The long chord C may be found by subtracting four ninths of the x correction from the length of spiral. Similarly the spiral tangent-distance v is found by adding one third of the x correction to one third of the length of spiral. Interpolations for distances between those given in the tables may be made, but it is best to compute R and the angles.

140. Laying out spiral.—The same methods may be used in laying out spirals of short radii as have been described for curves of large radii. The location of the P.S., P.C.C., and P.C. is generally not difficult. If the lines have been run to an intersection, as is generally desirable, the tangent-distance T may be measured to locate the P.S. The P.C.C. may be located by turning off the full spiral deflection angle θ_1. at the P.S. and measuring the long chord C; or the spiral tangent distances u and v may be calculated and the angle Δ_1 turned at their intersection point. In either case the tangent at the P.C.C. may readily be found. Another method is to locate the P.C. by offsetting the distance o from the initial tangent and then running in the circular curve to the P.C.C.

141. Centers for the spiral may be set (a) by measuring ordinates from the initial tangent for the full length of spiral; (b) by ordinates from the initial tangent as far as the middle of the spiral and from the produced circular curve for the remaining half length; (c) by ordinates from the initial tangent for about two thirds of the spiral and from the terminal spiral tangent for the remainder of the spiral. The offsets from the circular curve will be the same for given distances from the P.C.C. as the y for an equal length of spiral. The offsets from the terminal spiral tangent will be the difference between the offset for the circular curve and the y for a spiral, both for a length equal to the distance from P.C.C. to the point located. This will be $\frac{s^2}{2R}-\frac{s^2}{6k}$, where s is the distance of the point from the P.C.C. and R is the radius of the circular curve.

142. Location by means of deflection angles from initial tangent at P.S. is so similar to that for railway

spirals already described that it need not be further discussed. If the rails have previously been bent to their proper curvature very few centers need be set.

143. Arc excess.—It must be borne in mind that the actual length of arc is considered in the formulas here given, and care must be taken to provide for the difference between arc and chord measurement. The long chord is easily found by the x correction of the tables as already indicated. For other chords of spiral arcs (not from P.S.), it will be sufficiently accurate to use for the excess of arc over chord the excess of the same length of circular arc having a radius equal to that of the middle point of the spiral arc under consideration. The excess length of a circular arc over its chord may be calculated from the approximate formula $\frac{c^3}{24\ R^2}$, where c may be used as the length of either chord or arc. It may also be noted here that the number of degrees of angle in a circular arc is $\frac{57.3s}{R}$, and the deflection angle from tangent is of course half of this.

144. Curving rails.—The principles here outlined are for the center line of track, but it may be desirable to have measurements for the curves formed by the rails for use in curving rails, etc. Although the outer and inner rails will be parallel to the center line, their lines will not be true spirals, and allowance should be made for this. For data in bending rails, it will be well first to get the variation in length of rail from the length of the center line. For a given point on the center line, first find the Δ of the spiral. The outer rail will be $\frac{\Delta}{180}$ $\pi \times \frac{1}{2}G$ longer than the center line, and the inner rail will be as much shorter, G being the gauge of track.

Thus for $k = 1500$ and a spiral distance of 30 ft. $\Delta = 17°$ $11'$ and the excess length in outer rail is $\frac{17.18}{180} \times 3.14 \times 2.35 = .70$ ft. The outer rail distance will be 30.70 ft. and the inner rail distance 29.30 ft. The ordinate of the rail from its own initial tangent will be the y for the center line spiral $\pm \frac{1}{2} G$ vers Δ; plus to be used for the outer rail and minus for the inner one. Thus for the example above cited, the ordinates for the rails opposite a center distant 30 ft. from the P.S. (30.70 along outer rail and 29.30 ft. along the inner rail) will be $3.00 \pm .11 = 3.11$ and 2.89. In locating points on the rails, allowance should be made for the difference between the center line distance and the rail distance. The x for the point on the rail will be the x for the corresponding point on the center spiral $\pm \frac{1}{2} G \sin \Delta$. These principles will apply to any point on the spiral. It will be well to tabulate values for the sets of curves most used.

145. If it is desired to locate the last third of the spiral from the terminal spiral tangent for the two rails, the length and position of these tangents may be calculated from their ordinates and the Δ already used, and points on the rails located by offsets from these tangents. These offsets may readily be calculated by the principles already outlined.

146. Double track.—Double track curves will generally need radii of different lengths. The spirals for both curves may be taken from the same table, or the one for the outside curve may be taken from the table having a value of k next higher than that used with the inside curve. In the latter case the two spirals will be of nearly the same length and their ends will be nearly opposite. If the distance between center lines on the curve be made equal to the distance between center lines

on tangent plus the difference in the o for the inside and outside spiral, the circular parts of the two curves will be parallel and have the same center, and the radius of the outside curve will be equal to the radius of the inside curve plus the distance between tracks on the curve.

147. In case consideration of clearance requires greater distance between the tracks on curves than on tangents, care must be exercised in the selection of the spiral. The calculation of the external-distance E will best enable the distance between center lines at their middle points to be determined. If the inside radius is assumed, first find its external distance E, to this add the distance from one P. I. to the other, and subtract the required distance between the curves. The remainder will be the external-distance E for the outside curve from which the desired radius may be found. As by this arrangement the two curves will be closer at their ends than at their middle, care must be taken to secure sufficient clearance. The selection of curves and spirals for double track is more complex than for single track.

CONCLUSION

148. Besides the problems and methods here presented, many other applications may be made. For particular conditions the engineer may develop special methods.

The preceding methods generally have been based upon the principle that the spiral is to have the same degree-of-curve at the end as the main curve, and slight modifications may be necessary when not so. The value of o and of the angle in the circular curve omitted must be that for the spiral used. Thus with $a = 2$ the spiral at the

end of 300 ft. will be a 6° curve. It may, however, be there compounded with a curve of different radius, as a 6° 30′ curve, provided the offset is 3.93 and the central angle between the P.C. and the P.C.C. is 9°. Generally, in order to utilize the problems given for old track, etc., the formulas will need to be modified if D_0 does not agree with D_1.

149. In field work most of the usual formulas of the various location problems, like "Required to change the P.C. so that the curve may end in a parallel tangent," may be used without modification with curves having transition endings, by simply considering the whole intersection angle including the angle in the spirals. This is true whenever the same amount of spiral is used with the new curve.

If the degree-of-curve changes and with it the length of the spiral, the difference between the o's in the two cases must be allowed for. With a little practice in using such formulas with spirals, the engineer will find no difficulty.

150. The transition spiral has the merit of comparative simplicity and extreme flexibility. It is a natural method, since it is so similar to the methods used in laying out circular curves. Like circular curves, the length along the curve is the principal term, and the degree-of-curve, central angle, deflection angles and ordinates are obtainable from this variable. It may be used with any main curve, even of fractional degree; any length of chord may be used in measurement under the same restrictions as circular curves, and as it is not necessary to restrict the measurements to a common chord length, intermediate points may be readily located. The calculations for angles and distances are easily made. If desired, the tangent and the circular curve may be run out

and the spiral put in by co-ordinates, one half from the tangent and one half from the circular curve. This is especially applicable to location work and to short spirals.

The engineer should not be frightened by the mathematics in the demonstration of the formulas; the principles and methods may be understood without mastering the demonstrations. Experience has shown that the ordinary transit man, with a little thought and study, can understand and use the transition spiral as easily as circular curves, and that young assistants without previous training readily take up the work.

151. With reference to the use of the cubic parabola as an easement it may be said that, except for the relation between x and y, it has no properties of value for a transition curve which are not merely approximations of the transition spiral. Within small limits, the radius of curvature and the angles to be used approach somewhat closely to those for the transition spiral. As soon as x differs materially from the length of curve, a correction has to be made. The radius of the curvature finally begins to increase. The investigation of the cubic parabola in reference to its radius of curvature, its angle turned, the angular deflection to points on it, and the length of the curve, require as long mathematical equations as those governing the transition spiral. Many attempts have been made to utilize this curve, but both field work and computations are too intricate and inconvenient if the curve has any considerable length, and it has no advantage over the transition spiral.

152. The question of the efficiency of easement curves is of considerable importance. The objection is sometimes raised that even if track is laid out with a carefully fitted spiral there would be no possibility of keeping it in place by the methods of the ordinary trackman. This

identical objection could be made with the same force against carefully laid out circular curves, yet no engineer would recommend abolishing that practice. Even if, in re-lining, the transition curve is considerably distorted, it remains an easement, and will be in far better riding condition than a distorted circular curve. By marking the P.S. and the P.C.C with a stake or post, with intermediate points on long spirals, the trackman will be able to keep the spiral in as good condition as though it were of uniform curvature. The short spirals advocated by some engineers have proved to be insufficient. For efficient service, a length of spiral which will give an *o* of considerable amount must be used, even if this necessitates widening the roadbed.

153. Properly constructed spirals would frequently allow the use of sharper curvature—since the riding quality of curves may be the governing consideration in the selection of a maximum—and thus make a saving in construction. By fitting curves with proper transition spirals, roads using sharp curves may partially relieve the objection of the public to traveling by their routes. The introduction of fast trains has made it necessary to take every precaution to secure an easy-riding track. The disagreeable lurch and necessary "slow order" for fast trains at certain curves on many roads has been entirely eliminated by the construction of proper spirals, and passengers do not now know when such curves are reached. The transition curve has, then, a financial value largely overbalancing its cost. The adoption of such curves by many of our principal railways proves their efficiency, and the future will see a much more general adoption.

EXPLANATION OF TABLES.

In Tables I-XI, the columns give the following properties:

1. The distance in feet from the P.S. along the spiral to a point on the spiral; *i. e.,* 100 L. The full length of spiral will give values for the terminal point, the P.C.C. of main curve.

2. D, the degree-of-curve of the spiral at any point. It becomes D_1 at the P.C.C.

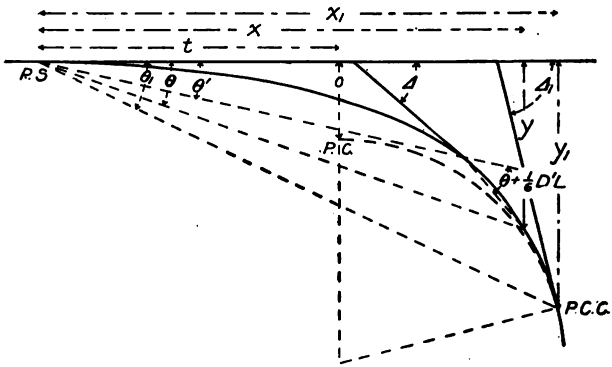

FIG. 17.

3. Δ, the spiral angle or change of direction of the spiral to the point.

4. θ, the spiral deflection angle at the P.S. from the initial tangent to locate the point.

5. o, the offset from the initial tangent to the P.C. of main curve produced backward. Enter the table with the full length of the spiral used.

6. y, the ordinate from the initial tangent as the axis of X.

7. x Cor., an amount to be subtracted from the distance in feet from the P.S. along the spiral to find the abscissa, x, of the point. $x = 100\ L - x$ Cor.

8. t Cor., an amount to be subtracted from half the full length of the spiral in feet to find t, the abscissa of the P.C. Enter the table with full length of the spiral used. $t = \frac{100}{2} L - t$ Cor.

To find the long chord to P.S., subtract four ninths (.444) of x Cor. from the length of the spiral in feet. $C = 100\ L - \frac{4}{9}\ x$ Cor. For chords not ending at P.S., see pages 16 and 77.

To find the terminal spiral tangent-distance, add one third x Cor. to one third the spiral distance to the point. $v = \frac{100}{3} L + \frac{1}{3}\ x$ Cor.

Intermediate values may be found by interpolation.

With transit at intermediate point on spiral, for deflection angle Φ, see pages 10, 20, and 40.

To use Table IV for other values of a, multiply the tabulated values of D, Δ, θ, o, and y in Table IV opposite the given distance from the P.S. by the a of the desired spiral, and x Cor. and t Cor. by the square of a. For inaccuracies of this method see page 27. If a and D or o are given, first find L.

Table XII permits ordinates to be calculated from o. See Fig. 5.

For the use of Tables XIII and XIV see page 67, and for Tables XV, XIX, see page 75.

Table XX gives values of o and L for values of a and D.

For full nomenclature, see page 3.

For equations and summary of principles, see page 17.

For fuller explanation of tables and errors of interpolation, see page 24.

For choice of a, see page 28.

TABLE I. TRANSITION SPIRAL.

1° in 200 ft. $a=\frac{1}{2}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
25	0° 07½′	0° 00′.9	0° 00′.3	.00	.00		
50	0 15	0 03.8	0 01.3	.00	.02		
75	0 22½	0 08.4	0 02.8	.02	.06		
100	0 30	0 15.	0 05.	.04	.15		
125	0 37½	0 23.4	0 07 8	.07	.29		
150	0 45	0 33.8	0 11.3	.12	.49		
175	0 52½	0 45.9	0 15.3	.20	.78	.00	
200	1 00	1 00.	0 20.	.29	1.16	.01	
225	1 07½	1 15.9	0 25.3	.41	1.66	.01	
250	1 15	1 33.8	0 31.3	.57	2.27	.02	.00
275	1 22½	1 53.4	0 37.8	.76	3.03	.03	.01
300	1 30	2 15.	0 45.	.98	3.93	.05	.01
325	1 37½	2 38.4	0 52.8	1.25	5.00	.07	.01
350	1 45	3 03.8	1 01.3	1.56	6.23	.10	.02
375	1 52½	3 30.9	1 10.3	1.92	7.67	.14	.02
400	2 00	4 00.	1 20.	2.33	9.31	.19	.03
425	2 07½	4 30.9	1 30.3	2.79	11.16	.26	.04
450	2 15	5 03.8	1 41.3	3.31	13.25	.35	.06
475	2 22½	5 38.4	1 52.8	3.89	15.58	.46	.08
500	2 30	6 15.	2 05.	4.54	18.16	.59	.10
525	2 37½	6 53.4	2 17.8	5.26	21.03	.75	.13
550	2 45	7 33.8	2 31.3	6.04	24.17	.95	.16
575	2 52½	8 15.9	2 45.3	6.91	27.62	1.20	.20
600	3 00	9 00.	3 00.	7.84	31.36	1.48	.24
625	3 07½	9 45.9	3 15.3	8.87	35.45	1.81	.30
650	3 15	10 33.8	3 31.3	9.97	39.85	2.21	.37
675	3 22½	11 23.4	3 47.8	11.16	44.63	2.66	.44
700	3 30	12 15.	4 04.9	12.45	49.73	3.20	.53
725	3 37½	13 08.4	4 22.7	13.83	55.22	3.81	.64
750	3 45	14 03.8	4 41.2	15.30	61.09	4.51	.75
775	3 52½	15 00.9	5 00.1	16.88	67.37	5.31	.89
800	4 00	16 00.	5 19.8	18.56	74.05	6 22	1.04

TABLE II. TRANSITION SPIRAL.

1° in 150 ft. $a=\frac{2}{3}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
25	0° 10′	0 01′.3	0 00′4	.00	.00		
50	0 20	0 05.	0 01.7	.01	.02		
75	0 30	0 11.3	0 03.8	.02	.08		
100	0 40	0 20.	0 06.7	.05	.19		
125	0 50	0 31.3	0 10.4	.10	.38		
150	1 00	0 45.	0 15.	.16	.65	0.00	
175	1 10	1 01.3	0 20.4	.26	1.04	0.01	
200	1 20	1 20	0 26.7	.39	1 55	0 01	

TABLE II.—*Continued.*

1° in 150 ft. $a=\frac{2}{3}$

Length	D	Δ	θ	o	y	x COR.	t COR.
225	1°30′	1°41′3	0°33′.8	.55	2.21	.02	.00
250	1 40	2 05.	0 41.7	.76	3 03	.03	.01
275	1 50	2 31.3	0 50.4	1 01	4.04	.05	.01
300	2 00	3 00.	1 00.	1.31	5.23	.08	.01
325	2 10	3 31.3	1 10.4	1.66	6.66	.12	.02
350	2 20	4 05.	1 21.7	2.08	8.31	.18	.03
375	2 30	4 41.3	1 33.8	2.56	10 23	.25	.04
400	2 40	5 20.	1 46.7	3.10	12.40	.35	.06
425	2 50	6 01.3	2 00.4	3.72	14 88	.47	.08
450	3 00	6 45.	2 15.	4.41	17.66	.62	.10
475	3 10	7 31.3	2 30.4	5.19	20 76	.82	.14
500	3 20	8 20.	2 46.7	6.05	24.20	1.06	.18
525	3 30	9 11.3	3 03.8	7.01	28.02	1.35	.22
550	3 40	10 05.	3 21.7	8.05	32.19	1.70	.28
575	3 50	11 01.3	3 40.4	9.20	36.78	2.12	.36
600	4 00	12 00.	3 59.9	10.45	41 76	2.63	.44

TABLE III. TRANSITION SPIRAL.

1° in 125 ft. $a=\frac{1}{2}$

Length	D	Δ	θ	o	y	x COR.	t COR.
25	0°12′	0°01½′	0 00½′	.00	.00		
50	0 24	0 06	0 02	.01	.03		
75	0 36	0 13½	0 04½	.02	.10		
100	0 48	0 24	0 08	.06	.23		
125	1 00	0 37½	0 12½	.11	.46		
150	1 12	0 54	0 18	.20	.79	.00	
175	1 24	1 13½	0 24½	.31	1.25	.01	
200	1 36	1 36	0 32	.47	1.86	.02	
225	1 48	2 01½	0 40½	.66	2.65	.03	.00
250	2 00	2 30	0 50	.91	3.64	.05	.01
275	2 12	3 01½	1 00½	1.21	4.84	.08	.01
300	2 24	3 36	1 12	1.57	6.28	.12	.02
325	2 36	4 13½	1 24½	2.00	7.99	.18	.03
350	2 48	4 54	1 38	2.49	9.97	.26	.04
375	3 00	5 37½	1 52½	3.07	12.27	.36	.06
400	3 12	6 24	2 08	3.72	14.88	.50	.08
425	3 24	7 13½	2 24½	4.47	17.85	.68	.11
450	3 36	8 06	2 42	5.31	21.18	.90	.15
475	3 48	9 01½	3 00½	6.23	24.90	1.18	.20
500	4 00	10 00	3 20	7.26	29.02	1.52	.25

TABLE V.—*Continued.*

1° in 80 ft. $a=1\frac{1}{4}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor
160	2°00′	1°36′	0°32′	.37	1.49	.0	.0
170	2 07½	1 48½	0 36	.45	1.77		
180	2 15	2 01½	0 40½	.53	2.12		
190	2 22½	2 15½	0 45	.62	2.50		
200	2 30	2 30	0 50	.73	2.90		
210	2 37½	2 45½	0 55	.84	3.36		
220	2 45	3 01½	1 00½	.97	3.87		
230	2 52½	3 18½	1 06	1.10	4.42		
240	3 00	3 36	1 12	1.25	5.02		
250	3 07½	3 54½	1 18	1.42	5.67	.1	
260	3 15	4 13½	1 24½	1.59	6.38	.1	
270	3 22½	4 33½	1 31	1.79	7.15	.2	
280	3 30	4 54	1 38	1.99	7.98	.2	
290	3 37½	5 15½	1 45	2.21	8.86	.2	
300	3 45	5 37½	1 52½	2.45	9.81	.3	
310	3 52½	6 00½	2 00	2.70	10.74	.3	
320	4 00	6 24	2 08	2.98	11.91	.4	
330	4 07½	6 48½	2 16	3.26	13.06	.4	
340	4 15	7 13½	2 24½	3.57	14.28	.5	
350	4 22½	7 39½	2 33	3.89	15.57	.6	
360	4 30	8 06	2 42	4.23	16.95	.7	.1
370	4 37½	8 33½	2 51	4.59	18.40	.8	.1
380	4 45	9 01½	3 00½	4.97	19.92	.9	.2
390	4 52½	9 30½	3 10	5.38	21.54	1.0	.2
400	5 00	10 00	3 20	5.80	23.23	1.2	.2
410	5 07½	10 30½	3 30	6.26	25.00	1.4	.2
420	5 15	11 01½	3 40½	6.72	26.86	1.6	.3
430	5 22½	11 33½	3 51	7.22	28.82	1.7	.3
440	5 30	12 06	4 02	7.74	30.87	2.0	.3
450	5 37½	12 39½	4 13	8.28	33.02	2.2	.4
460	5 45	13 13½	4 24½	8.84	35.25	2.4	.4
470	5 52½	13 48½	4 36	9.41	37.59	2.7	.5
480	6 00	14 24	4 48	10.03	40.02	3.0	.5
490	6 07½	15 00½	5 00	10.67	42.56	3.4	.6
500	6 15	15 37½	5 12½	11.33	45.20	3.7	.6
510	6 22½	16 15½	5 25	12.03	47.95	4.1	.7
520	6 30	16 54	5 38	12.74	50.79	4.5	.8
530	6 37½	17 33½	5 51	13.48	53.76	5.0	.8
540	6 45	18 13½	6 04	14.26	56.84	5.4	.9
550	6 52½	18 54½	6 18	15.07	60.02	6.0	1.0
560	7 00	19 36	6 32	15.90	63.34	6.5	1.1
570	7 07½	20 18½	6 46	16.76	66.72	7.1	1.2
580	7 15	21 01½	7 00	17.65	70.26	7.8	1.3
590	7 22½	21 45½	7 14½	18.57	73.90	8.4	1.4
600	7 30	22 30	7 29	19.52	77.68	9.2	1.5

TABLE IV.—*Continued.*

1° in 100 ft. $a=1$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
460	4.6°	10°34'.8	3°31'.6	7.07	28.24	1.57	.26
470	4.7	11 02.7	3 40.9	7.54	30.12	1.74	.29
480	4.8	11 31.2	3 50.4	8.03	32.07	1.94	.32
490	4.9	12 00.3	4 00.1	8.54	34.11	2.15	.36
500	5.0	12 30.	4 10.	9.07	36.23	2.37	.40
510	5.1	13 00.3	4 20.1	9.63	38.44	2.62	.44
520	5.2	13 31.2	4 30.4	10.20	40.73	2.89	.48
530	5.3	14 02.7	4 40.9	10.80	43.12	3.17	.53
540	5.4	14 34.8	4 51.4	11.42	45.59	3.49	.58
550	5.5	15 07.5	5 02.3	12.07	48.15	3.82	.64
560	5.6	15 40.8	5 13.4	12.74	50.83	4.18	.70
570	5.7	16 14.7	5 24.7	13.43	53.56	4.56	.76
580	5.8	16 49.2	5 36.2	14.14	56.40	4.98	.83
590	5.9	17 24.3	5 47.8	14.89	59.34	5.42	.90
600	6.0	18 00.	5 59.7	15.65	62.39	5.89	.98
610	6.1	18 36.3	6 11.8	16.44	65.52	6.40	1.07
620	6.2	19 13.2	6 24.1	17.26	68.77	6.94	1.16
630	6.3	19 50.7	6 36.5	18.10	72.11	7.51	1.25
640	6.4	20 28.8	6 49.1	18.97	75.56	8.13	1.36
650	6.5	21 07.5	7 02.0	19.87	79.11	8.78	1.47
660	6.6	21 47.	7 15.1	20.79	82.79	9.48	1.57
670	6.7	22 27.	7 28.5	21.74	86.56	10.22	1.69
680	6.8	23 07.	7 41.8	22.73	90.43	11.00	1.82
690	6.9	23 48.	7 55.3	23.73	94.42	11.82	1.96
700	7.0	24 30.	8 09.3	24.79	98.50	12.70	2.10

TABLE V. TRANSITION SPIRAL.

1° in 80 ft. $a=1\frac{1}{4}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°07½′	0°00½′	0°00′	.00	.00	.00	.00
20	0 15	0 01½	0 00½	.00	.00		
30	0 22½	0 03½	0 01	.00	.01		
40	0 30	0 06	0 02	.00	.02		
50	0 37½	0 09½	0 03	.01	.04		
60	0 45	0 13½	0 04½	.02	.08		
70	0 52½	0 18½	0 06	.03	.12		
80	1 00	0 24	0 08	.05	.19		
90	1 07½	0 30½	0 10	.07	.26		
100	1 15	0 37½	0 12½	.09	.36		
110	1 22½	0 45½	0 15	.12	.48		
120	1 30	0 54	0 18	.16	.63		
130	1 37½	1 03½	0 21	.20	.80		
140	1 45	1 13½	0 24½	.25	1.00		
150	1 52½	1 24½	0 28	.31	1.23	.00	.00

TABLE V.—*Continued.*

1° in 80 ft. $a=1\frac{1}{4}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor
160	2 00′	1° 36′	0° 32′	.37	1.49	.0	.0
170	2 07½	1 48½	0 36	.45	1.77		
180	2 15	2 01½	0 40½	.53	2.12		
190	2 22½	2 15½	0 45	.62	2.50		
200	2 30	2 30	0 50	.73	2.90		
210	2 37½	2 45½	0 55	.84	3.36		
220	2 45	3 01½	1 00½	.97	3.87		
230	2 52½	3 18½	1 06	1.10	4.42		
240	3 00	3 36	1 12	1.25	5.02		
250	3 07½	3 54½	1 18	1.42	5.67	.1	
260	3 15	4 13½	1 24½	1.59	6.38	.1	
270	3 22½	4 33½	1 31	1.79	7.15	.2	
280	3 30	4 54	1 38	1.99	7.98	.2	
290	3 37½	5 15½	1 45	2.21	8.86	.2	
300	3 45	5 37½	1 52½	2.45	9.81	.3	
310	3 52½	6 00½	2 00	2.70	10.74	.3	
320	4 00	6 24	2 08	2.98	11.91	.4	
330	4 07½	6 48½	2 16	3.26	13.06	.4	
340	4 15	7 13½	2 24½	3.57	14.28	.5	
350	4 22½	7 39½	2 33	3.89	15.57	.6	
360	4 30	8 06	2 42	4.23	16.95	.7	.1
370	4 37½	8 33½	2 51	4.59	18.40	.8	.1
380	4 45	9 01½	3 00½	4.97	19.92	.9	.2
390	4 52½	9 30½	3 10	5.38	21.54	1.0	.2
400	5 00	10 00	3 20	5.80	23.23	1.2	.2
410	5 07½	10 30½	3 30	6.26	25.00	1.4	.2
420	5 15	11 01½	3 40½	6.72	26.86	1.6	.3
430	5 22½	11 33½	3 51	7.22	28.82	1.7	.3
440	5 30	12 06	4 02	7.74	30.87	2.0	.3
450	5 37½	12 39½	4 13	8.28	33.02	2.2	.4
460	5 45	13 13½	4 24½	8.84	35.25	2.4	.4
470	5 52½	13 48½	4 36	9.41	37.59	2.7	.5
480	6 00	14 24	4 48	10.03	40.02	3.0	.5
490	6 07½	15 00½	5 00	10.67	42.56	3.4	.6
500	6 15	15 37½	5 12½	11.33	45.20	3.7	.6
510	6 22½	16 15½	5 25	12.03	47.95	4.1	.7
520	6 30	16 54	5 38	12.74	50.79	4.5	.8
530	6 37½	17 33½	5 51	13.48	53.76	5.0	.8
540	6 45	18 13½	6 04	14.26	56.84	5.4	.9
550	6 52½	18 54½	6 18	15.07	60.02	6.0	1.0
560	7 00	19 36	6 32	15.90	63.34	6.5	1.1
570	7 07½	20 18½	6 46	16.76	66.72	7.1	1.2
580	7 15	21 01½	7 00	17.65	70.26	7.8	1.3
590	7 22½	21 45½	7 14½	18.57	73.90	8.4	1.4
600	7 30	22 30	7 29	19.52	77.68	9.2	1.5

TABLE VI.—TRANSITION SPIRAL.

1° in 60 ft. $a = 1\frac{2}{3}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°10′	0°00½′	0°00′	.00	.00	.0	0
20	0 20	0 02	0 00½				
30	0 30	0 04½	0 01½				
40	0 40	0 08	0 03		.03		
50	0 50	0 12½	0 04		.06		
60	1 00	0 18	0 06	.03	.10		
70	1 10	0 24½	0 08	.04	.17		
80	1 20	0 32	0 10½	.06	.25		
90	1 30	0 40½	0 13½	.09	.35		
100	1 40	0 50	0 16½	.12	.48		
110	1 50	1 00½	0 20	.16	.64		
120	2 00	1 12	0 24	.21	.84		
130	2 10	1 24½	0 28	.26	1.06		
140	2 20	1 38	0 32½	.33	1.33		
150	2 30	1 52½	0 37½	.41	1.63		
160	2 40	2 08	0 42½	.50	1.98		
170	2 50	2 24½	0 48	.59	2.38		
180	3 00	2 42	0 54	.70	2.82		
190	3 10	3 00½	1 00	.83	3.32		
200	3 20	3 20	1 06½	.97	3.88		
210	3 30	3 40½	1 13½	1.12	4.48	.1	
220	3 40	4 02	1 20½	1.29	5.15	.1	
230	3 50	4 24½	1 28	1.47	5.90	.1	
240	4 00	4 48	1 36	1.67	6.69	.2	
250	4 10	5 12½	1 44	1.89	7.58	.2	
260	4 20	5 38	1 52½	2.13	8.52	.2	
270	4 30	6 04½	2 01½	2.38	9.54	.3	
280	4 40	6 32	2 10½	2.65	10.64	.4	
290	4 50	7 00½	2 20	2.94	11.82	.4	
300	5 00	7 30	2 30	3.26	13.07	.5	
310	5 10	8 00½	2 40	3.60	14.43	.6	.1
320	5 20	8 32	2 50½	3.96	15.87	.7	.1
330	5 30	9 04½	3 01½	4.34	17.40	.8	.1
340	5 40	9 38	3 12½	4.75	19.02	.9	.2
350	5 50	10 12½	3 24	5.18	20.74	1.1	.2
360	6 00	10 48	3 36	5.64	22.56	1.3	.2
370	6 10	11 24½	3 48	6.12	24.50	1.4	.2
380	6 20	12 02	4 00½	6.63	26.53	1.7	.3
390	6 30	12 40½	4 13½	7.16	28.67	1.9	.3
400	6 40	13 20	4 26½	7.73	30.92	2.2	.4
410	6 50	14 00½	4 40	8.34	33.27	2.4	.4
420	7 00	14 42	4 54	8.96	35.73	2.8	.5
430	7 10	15 24½	5 08	9.61	38.32	3.1	.5
440	7 20	16 08	5 22½	10.30	41.07	3.5	.6
450	7 30	16 52½	5 37½	11.01	43.90	3.9	.6

TABLE VI.—*Continued.*

1° in 60 ft. $a=1\frac{2}{3}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
460	7°40′	17°38′	5°52′	11.75	46.86	4.3	.7
470	7 50	18 24½	6 08	12.50	49.94	4.8	.8
480	8 00	19 12	6 24	13.35	53.16	5 4	.9
490	8 10	20 00½	6 40	14.19	56.52	5.9	1.0
500	8 20	20 50	6 56	15.07	60.01	6.6	1.1
510	8 30	21 40½	7 13	16.00	63.64	7.2	1.2
520	8 40	22 32	7 30	16.94	67.36	8.0	1.3
530	8 50	23 24½	7 47½	17.93	71.25	8.8	1.5
540	9 00	24 18	8 05	18.95	75.31	9 6	1.6
550	9 10	25 12½	8 23	20.03	79.53	10.5	1.8
560	9 20	26 08	8 42	21.13	83.88	11.5	1.9
570	9 30	27 04½	9 00½	22.26	88.31	12.6	2.1
580	9 40	28 02	9 19½	23.42	92.92	13.7	2.3
590	9 50	29 00½	9 39	24.67	97.70	14.9	2.5
600	10 00	30 00	9 59	25.91	102.66	16.2	2.7

TABLE VII. TRANSITION SPIRAL.

1° in 50 ft. $a=2$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°12′	0°00½′	0°00′	.00	.00	.0	.0
20	0 24	0 02½	0 01				
30	0 36	0 05½	0 02		.02		
40	0 48	0 09½	0 03	.01	.04		
50	1 00	0 15	0 05	.02	.07		
60	1 12	0 21½	0 07	.03	.13		
70	1 24	0 29½	0 10	.05	.20		
80	1 36	0 38½	0 13	.07	.30		
90	1 48	0 48½	0 16	.10	.42		
100	2 00	1 00	0 20	.15	.58		
110	2 12	1 12½	0 24	.19	.77		
120	2 24	1 26½	0 29	.25	1.00		
130	2 36	1 41½	0 34	.32	1.28		
140	2 48	1 57½	0 39	.40	1 60		
150	3 00	2 15	0 45	.49	1.96		
160	3 12	2 33½	0 51	.59	2.38		
170	3 24	2 53½	0 58	.71	2 86		
180	3 36	3 14½	1 05	.85	3.39	.1	
190	3 48	3 36½	1 12	1.00	3 99	.1	
200	*4 00*	4 00	1 20	1.16	4 65	.1	.0

TABLE IX. TRANSITION SPIRAL.

1° in 30 ft. $a=3\frac{1}{3}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°20′	0°01′	0°00′	.00	.00	.0	.0
20	0 40	0 04	0 01	.00	.01		
30	1 00	0 09	0 03	.01	.03		
40	1 20	0 16	0 05	.02	.06		
50	1 40	0 25	0 08	.03	.12		
60	2 00	0 36	0 12	.05	.21		
70	2 20	0 49	0 16	.08	.33		
80	2 40	1 04	0 21	.12	.50		
90	3 00	1 21	0 27	.18	.71		
100	3 20	1 40	0 33	.24	.97		
110	3 40	2 01	0 40	.32	1.29		
120	4 00	2 24	0 48	.42	1.68		
130	4 20	2 49	0 56	.53	2.13		
140	4 40	3 16	1 05	.67	2.66		
150	5 00	3 45	1 15	.82	3.27	.1	
160	5 20	4 16	1 25	.99	3.97	.1	
170	5 40	4 49	1 36	1.19	4.76	.1	
180	6 00	5 24	1 48	1.41	5.65	.2	
190	6 20	6 01	2 00	1.66	6.65	.2	
200	6 40	6 40	2 13	1.94	7.75	.3	
210	7 00	7 21	2 27	2.24	8.97	.3	.1
220	7 20	8 04	2 41	2.58	10.31	.4	.1
230	7 40	8 49	2 56	2.95	11.77	.5	.1
240	8 00	9 36	3 12	3.35	13.38	.7	.1
250	8 20	10 25	3 28	3.78	15.11	.8	.1
260	8 40	11 16	3 45	4.25	17.00	1.0	.2
270	9 00	12 09	4 03	4.76	19.02	1.2	.2
280	9 20	13 04	4 21	5.31	21.20	1.4	.2
290	9 40	14 01	4 40	5.90	23.55	1.7	.3
300	10 00	15 00	5 00	6.53	26.05	2.0	.3
310	10 20	16 01	5 20	7.20	28.72	2.4	.4
320	10 40	17 04	5 41	7.92	31.57	2.8	.5
330	11 00	18 09	6 03	8.69	34.59	3.3	.5
340	11 20	19 16	6 25	9.49	37.80	3.8	.6
350	11 40	20 25	6 48	10.35	41.19	4.4	.7
360	12 00	21 36	7 11	11.25	44.78	5.1	.8
370	12 20	22 49	7 36	12.21	48.56	5.8	1.0
380	12 40	24 04	8 00	13.22	52.53	6.6	1.1
390	13 00	25 21	8 26	14.28	56.71	7.6	1.3
400	13 20	26 40	8 52	15.39	61.10	8.6	1.4
410	13 40	28 01	9 19	16.56	65.69	9.7	1.6
420	14 00	29 24	9 47	17.79	70.49	10.9	1.8
430	14 20	30 49	10 15	19.07	75.51	12.3	2.1
440	14 40	32 16	10 43	20.41	80.74	13.7	2.3
450	15 00	33 45	11 13	21.81	86.19	15.4	2.6

TABLE VIII. TRANSITION SPIRAL.

1° in 40 ft. $a=2\frac{1}{2}$

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°15′	0°01′	0°00′	.00	.00	.0	.0
20	0 30	0 03	0 01				
30	0 45	0 07	0 02		.02		
40	1 00	0 12	0 04	.01	.05		
50	1 15	0 19	0 06	.02	.09		
60	1 30	0 27	0 09	.04	.16		
70	1 45	0 37	0 12	.06	.25		
80	2 00	0 48	0 16	.09	.37		
90	2 15	1 01	0 20	.13	.53		
100	2 30	1 15	0 25	.18	.73		
110	2 45	1 31	0 30	.24	.97		
120	3 00	1 48	0 36	.31	1.25		
130	3 15	2 07	0 42	.40	1.60		
140	3 30	2 27	0 49	.50	2.00		
150	3 45	2 49	0 56	.61	2.45		
160	4 00	3 12	1 04	.74	2.97		
170	4 15	3 37	1 12	.89	3.57		
180	4 30	4 03	1 21	1.06	4.24	.1	
190	4 45	4 31	1 30	1.25	4.99	.1	
200	5 00	5 00	1 40	1.45	5.81	.2	
210	5 15	5 31	1 50	1.68	6.72	.2	
220	5 30	6 03	2 01	1.93	7.74	.2	
230	5 45	6 37	2 12	2.20	8.85	.3	
240	6 00	7 12	2 24	2.51	10.05	.4	
250	6 15	7 49	2 36	2.84	11.37	.5	.1
260	6 30	8 27	2 49	3.19	12.77	.6	.1
270	6 45	9 07	3 02	3.57	14.29	.7	.1
280	7 00	9 48	3 16	3.98	15.94	.8	.1
290	7 15	10 31	3 30	4.42	17.70	1.0	.2
300	7 30	11 15	3 45	4.89	19.59	1.2	.2
310	7 45	12 01	4 00	5.40	21.61	1.4	.2
320	8 00	12 48	4 16	5.94	23.76	1.6	.3
330	8 15	13 37	4 32	6.51	26.05	1.9	.3
340	8 30	14 27	4 49	7.12	28.46	2.2	.4
350	8 45	15 19	5 06	7.77	31.03	2.5	.4
360	9 00	16 12	5 24	8.46	33.74	2.9	.5
370	9 15	17 07	5 42	9.18	36.62	3.3	.5
380	9 30	18 03	6 01	9.95	39.64	3.7	.6
390	9 45	19 01	6 20	10.75	42.82	4.3	.7
400	10 00	20 00	6 40	11.60	46.16	4.9	.8
410	10 15	21 01	7 00	12.47	49.65	5.5	.9
420	10 30	22 03	7 21	13.39	53.28	6.2	1.0
430	10 45	23 07	7 42	14.38	57.10	6.9	1.2
440	11 00	24 12	8 04	15.39	61.12	7.8	1.3
450	11 15	25 19	8 26	16.45	65.32	8.7	1.5

TABLE IX. TRANSITION SPIRAL.

1° in 30 ft. $a=3\frac{1}{3}$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°20′	0°01′	0°00′	.00	.00	.0	.0
20	0 40	0 04	0 01	.00	.01		
30	1 00	0 09	0 03	.01	.03		
40	1 20	0 16	0 05	.02	.06		
50	1 40	0 25	0 08	.03	.12		
60	2 00	0 36	0 12	.05	.21		
70	2 20	0 49	0 16	.08	.33		
80	2 40	1 04	0 21	.12	.50		
90	3 00	1 21	0 27	.18	.71		
100	3 20	1 40	0 33	.24	.97		
110	3 40	2 01	0 40	.32	1.29		
120	4 00	2 24	0 48	.42	1.68		
130	4 20	2 49	0 56	.53	2.13		
140	4 40	3 16	1 05	.67	2.66		
150	5 00	3 45	1 15	.82	3.27	.1	
160	5 20	4 16	1 25	.99	3.97	.1	
170	5 40	4 49	1 36	1.19	4.76	.1	
180	6 00	5 24	1 48	1.41	5.65	.2	
190	6 20	6 01	2 00	1.66	6.65	.2	
200	6 40	6 40	2 13	1.94	7.75	.3	
210	7 00	7 21	2 27	2.24	8.97	.3	.1
220	7 20	8 04	2 41	2.58	10.31	.4	.1
230	7 40	8 49	2 56	2.95	11.77	.5	.1
240	8 00	9 36	3 12	3.35	13.38	.7	.1
250	8 20	10 25	3 28	3.78	15.11	.8	.1
260	8 40	11 16	3 45	4.25	17.00	1.0	.2
270	9 00	12 09	4 03	4.76	19.02	1.2	.2
280	9 20	13 04	4 21	5.31	21.20	1.4	.2
290	9 40	14 01	4 40	5.90	23.55	1.7	.3
300	10 00	15 00	5 00	6.53	26.05	2.0	.3
310	10 20	16 01	5 20	7.20	28.72	2.4	.4
320	10 40	17 04	5 41	7.92	31.57	2.8	.5
330	11 00	18 09	6 03	8.69	34.59	3.3	.5
340	11 20	19 16	6 25	9.49	37.80	3.8	.6
350	11 40	20 25	6 48	10.35	41.19	4.4	.7
360	12 00	21 36	7 11	11.25	44.78	5.1	.8
370	12 20	22 49	7 36	12.21	48.56	5.8	1.0
380	12 40	24 04	8 00	13.22	52.53	6.6	1.1
390	13 00	25 21	8 26	14.28	56.71	7.6	1.3
400	13 20	26 40	8 52	15.39	61.10	8.6	1.4
410	13 40	28 01	9 19	16.56	65.69	9.7	1.6
420	14 00	29 24	9 47	17.79	70.49	10.9	1.8
430	14 20	30 49	10 15	19.07	75.51	12.3	2.1
440	14 40	32 16	10 43	20.41	80.74	13.7	2.3
450	15 00	33 45	11 13	21.81	86.19	15.4	2.6

TABLE X. TRANSITION SPIRAL.

1° in 20 ft. $a=5$.

Length	D	Δ	θ	o	y	x Cor.	t Cor.
10	0°30′	0°01′	0°00′	.00	.00	.0	.0
20	1 00	0 06	0 02		.01		
30	1 30	0 13	0 04	.01	.04		
40	2 00	0 24	0 08	.02	.09		
50	2 30	0 37	0 12	.05	.18		
60	3 00	0 54	0 18	.08	.31		
70	3 30	1 13	0 24	.12	.50		
80	4 00	1 36	0 32	.19	.74		
90	4 30	2 01	0 40	.26	1.06		
100	5 00	2 30	0 50	.36	1.45		
110	5 30	3 01	1 00	.48	1.94		
120	6 00	3 36	1 12	.62	2.51		
130	6 30	4 13	1 24	.79	3.20		
140	7 00	4 54	1 38	.99	3.99	.1	
150	7 30	5 37	1 52	1.22	4.90	.1	
160	8 00	6 24	2 08	1.48	5.96	.2	
170	8 30	7 13	2 24	1.78	7.15	.3	
180	9 00	8 06	2 42	2.11	8.49	.4	
190	9 30	9 01	3 00	2.49	9.98	.5	
200	10 00	10 00	3 20	2.90	11.62	.6	.1
210	10 30	11 01	3 40	3.36	13.45	.8	.1
220	11 00	12 06	4 02	3.86	15.44	1.0	.2
230	11 30	13 13	4 24	4.41	17.63	1.2	.2
240	12 00	14 24	4 48	5.01	20.01	1.5	.3
250	12 30	15 37	5 12	5.66	22.60	1.8	.3
260	13 00	16 54	5 38	6.37	25.38	2.2	.4
270	13 30	18 13	6 04	7.12	28.39	2.7	.5
280	14 00	19 36	6 32	7.94	31.62	3.3	.6
290	14 30	21 02	7 00	8.82	35.10	3.9	.7
300	15 00	22 30	7 29	9.76	38.83	4.6	.8
310	15 30	24 02	8 00	10.76	42.73	5.4	.9
320	16 00	25 36	8 31	11.82	46.92	6.3	1.1
330	16 30	27 13	9 04	12.95	51.36	7.4	1.2
340	17 00	28 54	9 37	14.15	56.05	8.6	1.4
350	17 30	30 37	10 11	15.43	61.09	9.9	1.7
360	18 00	32 24	10 46	16.75	66.31	11.3	1.9
370	18 30	34 14	11 19	18.16	71.63	13.0	2.2
380	19 00	36 06	12 00	19.65	77.35	14.8	2.5
390	19 30	38 02	12 38	21.21	83.41	16.8	2.8
400	20 00	40 00	13 17	22.87	89.83	19.0	3.2

TABLE XIV. COEFFICIENTS FOR DEFLECTION ANGLES.

To find the deflection angle from tangent at the instrument point, multiply the spiral deflection angle at the P. S. for a single chord length by the coefficient found in the instrument point column opposite the number of the chord point to be located.

Chord Point Number	Instrument at Chord Point Number																Chord Point Number
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
0	0	2	8	18	32	50	72	98	128	162	200	242	288	338	392	450	0
1	1	0	5	14	27	44	65	90	119	152	189	230	275	324	377	434	1
2	4	4	0	8	20	36	56	80	108	140	176	216	260	308	360	416	2
3	9	10	7	0	11	26	45	68	95	126	161	200	243	290	341	396	3
4	16	18	16	10	0	14	32	54	80	110	144	182	224	270	320	374	4
5	25	28	27	22	13	0	17	38	63	92	125	162	203	248	297	350	5
6	36	40	40	36	28	16	0	20	44	72	104	140	180	224	272	324	6
7	49	54	55	52	45	34	19	0	23	50	81	116	155	198	245	296	7
8	64	70	72	70	64	54	40	22	0	26	56	90	128	170	216	266	8
9	81	88	91	90	85	76	63	46	25	0	29	62	99	140	185	234	9
10	100	108	112	112	108	100	88	72	52	28	0	32	68	103	152	200	10
11	121	130	135	136	133	126	115	100	81	58	31	0	35	74	117	164	11
12	144	154	160	162	160	154	141	130	112	96	64	34	0	38	80	126	12
13	169	180	187	190	189	184	175	162	145	124	99	70	37	0	41	86	13
14	196	208	216	220	220	216	208	196	180	160	136	108	76	40	0	44	14
15	225	238	247	252	253	250	243	232	217	198	175	148	117	82	43	0	15

TABLE XV. STREET RAILWAY SPIRAL.

k=2000. k=2

Length	Radius	Δ	θ	o	y	x Cor.	t C
5	400	0°21′	0°07′		.01	.00	.
10	200	1 26	0 29		.08	.00	
15	133.33	3 13	1 04		.28	.00	
16	125	3 40	1 13		.34	.01	
20	100	5 44	1 55	.17	.67	.02	
21.06	95	6 21	2 07	.19	.78	.03	
22.22	90	7 04	2 21	.23	.91	.03	.
23.53	85	7 56	2 39	.27	1.09	.04	.
25	80	8 57	2 59	.32	1.30	.06	.
26.67	75	10 11	3 24	.39	1.58	.08	.
28.57	70	11 41	3 54	.48	1.94	.12	.
30	66.67	12 54	4 18	.56	2.24	.15	.
30.77	65	13 34	4 31	.61	2.42	.17	.
33 33	60	15 55	5 18	.77	3.07	.26	.
36.36	55	18 56	6 19	1 00	3.98	.40	.
40	50	22 55	7 38	1 32	5 27	.64	.
44.44	45	28 17	9 25	1.81	7.19	1.08	.
50	40	35 49	11 54	2.56	10.13	1.95	.
55	36.36	43 20	14 23	3.39	13.29	3 15	.
56.05	35.68	45 00	14 55	3.59	14.02	3 46	.

TABLE XVI. STREET RAILWAY SPIRAL.

k=1500. k=1

Length	Radius	Δ	θ	o	y	x Cor.	t C
5	300	0°29′	0°10′		.01	.00	.
10	150	1 55	0 38		.11	.00	
12	125	2 45	0 55		.19	.00	
15	100	4 18	1 26		.38	.01	
15.79	95	4 46	1 35		.43	.01	
16.67	90	5 19	1 46		.51	.01	
17.65	85	5 58	1 59		.61	.02	
18.75	80	6 43	2 14	.18	.73	.03	
20	75	7 38	2 33	.22	.89	.03	.
21.43	70	8 46	2 55	.27	1.09	.05	.
23.08	65	10 10	3 23	.34	1.36	.07	.
25	60	11 56	3 59	.43	1.74	.11	.
27.27	55	14 13	4 44	.56	2.24	.17	.
30	50	17 11	5 44	.75	2.98	.27	.
33.33	45	21 13	7 04	1.02	4.07	.45	.
35	42 86	23 24	7 47	1.18	4.71	.57	.
37.50	40	26 51	8 56	1.45	5.77	.82	.
40	37.50	30 34	10 10	1.76	6.96	1.14	.
42.86	35	35 05	11 40	2.16	8.51	1.61	.
45	33.33	38 41	12 51	2.49	9.80	2.05	.

TABLE XVII. STREET RAILWAY SPIRAL.

$k=1250$ $k=1250$

Length	Radius	Δ	θ	o	y	x Cor.	t Cor.
5	250	0°34′	0°11′		.02	.00	.00
10	125	2 17	0 46		.13		
15	83.33	5 11	1 44		.45	.01	
15.62	80	5 36	1 52		.51	.01	
16.67	75	6 22	2 07	.15	.62	.02	
17.86	70	7 19	2 26	.19	.76	.03	.00
19.23	65	8 29	2 50	.24	.95	.04	.01
20	62.50	9 10	3 03	.26	1.04	.05	.01
20.83	60	9 56	3 19	.30	1.20	.06	.01
22.73	55	11 50	3 57	.39	1.56	.10	.02
25	50	14 19	4 46	.52	2 07	.15	.02
27.78	45	17 41	5 54	.71	2.84	26	.04
30	41 66	20 38	6 52	.89	3.56	38	.06
31.25	40	22 24	7 28	1.01	5.04	.47	.08
35	35.71	28 05	9 20	1.40	5.62	.83	.14

TABLE XVIII. STREET RAILWAY SPIRAL.

$k=1000$ $k=1000$

Length	Radius	Δ	θ	o	y	x Cor.	t Cor.
5	200	0°43′	0°14′		.02	.00	.00
10	100	2 52	0 57		.17	.01	
15	66.67	6 27	2 09		.56	.02	
15.39	65	6 47	2 16	.15	.61	.02	
16.67	60	7 57	2 39	.19	.77	.03	.00
18.18	55	9 28	3 09	.25	1.00	.05	.01
20	50	11 27	3 49	.33	1 33	.08	.01
22.22	45	14 09	4 44	.46	1.83	.13	.02
25	40	17 54	5 57	.64	2.58	.24	.04
28.57	35	23 23	7 47	.97	3.84	.47	.08
30	33.33	25 47	8 34	1.12	4.45	.60	.10
33.33	30	31 50	10 35	1.53	6.05	1.03	.17

TABLE XIX. STREET RAILWAY SPIRAL.

$k=750$ $k=750$

Length	Radius	Δ	θ	o	y	x Cor.	t Cor.
5	150	0°57′	0°19′		.03	.00	.00
10	75	3 49	1 16		.22	.01	
15	50	8 54	2 52	.19	.75	.03	.00
16.67	45	10 37	3 32	.26	1.03	.06	.01
18.75	40	13 24	4 28	.36	1.46	.10	.02
20	37.5	15 17	5 05	.44	1.75	.14	.02
21.43	35	17 32	5 50	.57	2.27	.20	.03
25	30	23 52	7 57	.86	3.43	.43	.07